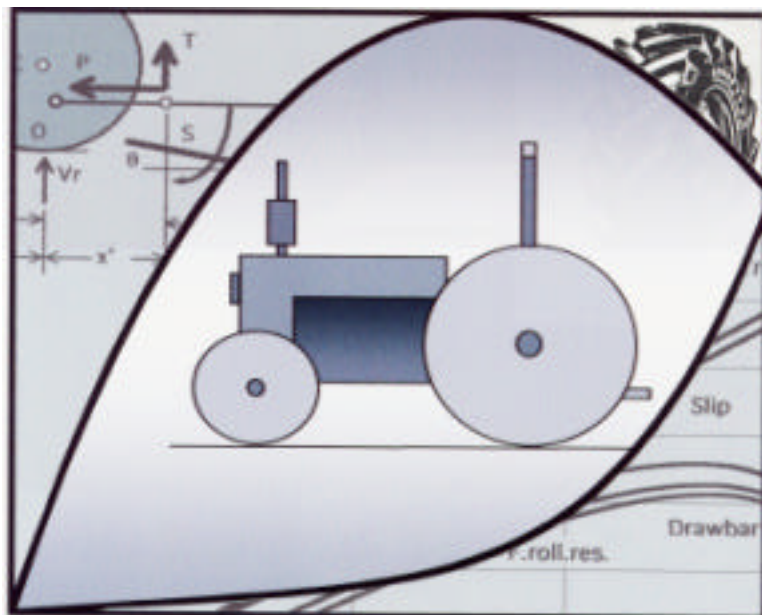


# **The Mechanics of Tractor - Implement Performance**

*Theory and Worked Examples*

**A TEXTBOOK FOR STUDENTS AND ENGINEERS**



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*Printed from: <http://www.eprints.unimelb.edu.au>*

## **Dedication**

*For my parents . . .*

## **Philosophy**

*‘To the writer however, the most important reason for the study of soil - vehicle mechanics is an educational one. The training of agricultural engineers at University level is a relatively new enterprise which aims at producing creative engineers in a shorter overall period than the old method of practical experience alone. In order to achieve this the University must concentrate on the teaching of principles and the scientific method applied to each particular field. The young engineer must then add to this some years of experience of the application of these principles and must support them with adequate background knowledge.*

*If the scientific approach is the aim of academic agricultural engineering, then it is plain that the principles of soil vehicle mechanics (and soil implement mechanics) must form an important part of the teaching. Unfortunately in this, as in other branches of agricultural engineering, the principles are obscure and can only be taught after considerable research on the part of the teacher. The research effort . . . is not aimed at the direct improvement of the farm tractor but rather at the elucidation of principles which can be taught to students who will use them in the development of better machines.’*

**A.R. Reece**

## **Prayer**

*I offer you tonight, Lord, the work of all the tractors . . . in the world.*

**Prayers of Life: Michel Quoist**

## PREFACE

This book arose out of the experience that the author has had in teaching courses on tractor performance for a number of years particularly at the University of Melbourne. It has been written primarily for student use in agricultural and mechanical engineering courses at University and College level and as such, it assumes:

- (a) a knowledge of basic mechanics, stress analysis, soil mechanics and power transmission elements appropriate to second year professional engineering courses;
- (b) a general knowledge of the layout and operation of the tractor.

The need for such a book arose out of the fact that, while there are other books written on the general topic of the agricultural tractor, none treat the subject of tractor performance in an adequate way that builds on the engineering science which is covered in first and second year engineering courses. Existing books tend to be too broad, being written to cover the whole subject from the design of engine components to the economics of use. Others, that are written essentially for users, merely describe the tractor and its operation. Nor is there a book written that provides an suitable background for general engineers wishing to 'break into' the technical or research literature.

In writing this book an attempt has been made to keep the discussion as general as possible. It is concerned with principles and does not become involved in consideration of the details of individual types of tractor even to the point of not distinguishing between two wheel (walking) and four wheel tractors (except in relation to chassis mechanics).

Further no attempt has been made to describe the construction of the tractor or its various components and operational systems. For those who wish to learn these details, reference should be made to the engineering textbooks specifically written on these topics and other books on the agricultural tractor that includes them.

The understanding of the concepts on which a book such as this is based owes much to many others who have published material on this subject; the author gratefully acknowledges the material that others have contributed in this way. However, two people and their associated groups must be mentioned in particular.

The first is the late G.H. Vasey and his colleagues at the University of Melbourne. Their development of the graphical representation of tractor performance (on which Chapter 3 is based) still provides the clearest understanding of the subject for students and others who would learn from it.

The second is A.R. Reece and his colleagues at the University of Newcastle-on-Tyne, England. Chapter 4 which is largely based on their work (and earlier work by Bekker) provides an understanding of the traction process in terms of engineering fundamentals that are suitable for use at the student level. Indeed the educational philosophy as presented by Reece (1964) on the dedication page seems entirely appropriate for this work.

The demise of agricultural engineering courses in developed countries and the need for cheap, basic educational materials in developing countries prompted the compilation of this work. Its publication on the University of Melbourne web site makes it available to a wide range of readers at little cost; it is hoped that, like the author, they will appreciate this facility!

The author also wishes to acknowledge the support of his colleagues, in particular the secretarial assistance of Ms. J. Wise, the comment on the text by Dr. Nguyen Phu Thien and the assistance in arranging for its publication on the University of Melbourne web site by Dr. Graham Moore. The support of the Universities of Melbourne, Australia and Hohenheim, Germany in providing the opportunity for study leave, during which much of the final compilation of the work took place, is also acknowledged.

The encouragement and help of his wife Joan in the checking the manuscript and in many other ways is cause for gratitude.

The author would value notification of any errors in this work.

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## CHAPTER 1

### THE AGRICULTURAL TRACTOR

#### 1.1 INTRODUCTION

##### 1.1.1 General

The agricultural tractor is one of the class of mobile machines that involves the 'traction' process. The word 'traction' and name 'tractor' come from the word to 'draw' or 'pull' so a tractor is basically a machine for pulling; other mobile machines such as locomotives are in the same class. Vehicles like road trucks and even motor cars, which are essentially vehicles for carrying loads, also involve the traction process.

The tractor is also in the class of machines that involves operation under what are known as 'off-road' conditions. Others in this class include machines used in earth moving, mining and military work, also four-wheel drive motor vehicles for cross - country operation.

##### 1.1.2 Justification

The question is often asked as to what is so special about the tractor and its operation that would justify its study as a machine in its own right. This may be answered by considering the conditions under which the tractor is expected operate.

- (i) The agricultural soils, on which the tractor operates, are 'weak', ie, they slip (shear) when loaded horizontally and compact (compress) when loaded vertically. This condition, which the tractor and its attached implement are frequently being used to produce, is usually ideal from an agricultural point of view but is not conducive to efficient operation from a tractive point of view.
- (ii) The loading conditions on the tractor are variable from job to job and, for efficient operation, ideally require the tractor to be set up to suit each condition.
- (iii) The operating conditions for the tractor are highly variable both in time and place, which requires continual monitoring and adjustment of both tractor and implement in operation.
- (iv) The ground surfaces are rough and sloping, hence both tractor and implement control is difficult; instability is an ever-present danger. This is important because the tractor must be able to be operated by non-specialists.
- (v) A clearance above growing crops and the ability for the operator to see the ground.

The tractor must function effectively and efficiently while satisfying these often conflicting requirements. The study of the tractive processes on soft soils and the dynamics of implement control, are unique to the agricultural tractor and justify specialized analysis, research and design. The present work builds on elementary aspects of the published literature on these studies and seeks to provide a basis for 'breaking into' the technical and research literature.

##### 1.1.3 Development

The tractor evolved in the second half of the 19th century and first half of the 20th into its present, conventional, two wheel drive form and four wheel drive variation. This form owes much to history but also the fact that it is an inherently logical arrangement.

- (i) Designers followed early tractor designs that were simply replacements for horses or other draught animals.
- (ii) The layout takes advantage of the transfer of weight to the main driving wheels at the rear, as the drawbar pull on the tractor increases.
- (iii) The layout is inherently stable in the horizontal plane because the implement commonly being pulled behind the tractor tends to follow the latter and to pull it into straight line operation.
- (iv) Rear mounted implements offer a minimum of offset loading and moment in the horizontal plane; this contrasts with, for example side mounted implements.



As a result there has been little or no major change in the basic lay-out of tractor / implement systems over their period of development although there have been major improvements in engines, transmissions, tyres, control systems and drivers' accommodation.

#### 1.1.4 Classification of types

Tractors may be classified according to their basic form, which in turn depends on the function that each type is designed to achieve. They may be classified as follows.

- |       |                                    |                                                                        |
|-------|------------------------------------|------------------------------------------------------------------------|
| (i)   | Number of axles                    | * one - walking<br>* two - conventional, riding                        |
| (ii)  | Number of driven axles             | * one - conventional and walking<br>* two - four wheel drive           |
| (iii) | Ground drive elements <sup>1</sup> | * wheels and tyres, lugs, strakes<br>* tracks - crawler, track laying  |
| (iv)  | Use of wheels                      | * traction - conventional<br>* propulsion / cultivation - power tiller |

Illustrations and descriptions of the various forms of tractor and the associated terminology may be found in other textbooks (Liljedahl et al (1989)).

### 1.2 FUNCTIONAL REQUIREMENTS AND LIMITATIONS

#### 1.2.1 Functional requirements

Although it is able to undertake a multitude of specific tasks, the functions of the tractor can be reduced to the following (Reece 1971):

- (i) the provision of up to full power in the form of a large drawbar pull (compared to the weight of the tractor) at low speeds. The highly variable loading that occurs in agricultural work requires consideration of tractor performance at part load, particularly with respect to fuel consumption.
- (ii) the provision of power for driving and control of a range of implements and machines performing various tasks and attached in a variety of ways.
- (iii) the provision of power as the basis for a transport system in both on- and off-road conditions.

The main emphasis in this book is on how the tractor performs these functions, ie, on its functional performance. There are of course other ways by which tractors might be evaluated such as by their economy, reliability, safety or ease of operation. These are important but are beyond the scope of this book.

#### 1.2.2 Performance limitations

Since its main function is to pull (or push), the question arises as to how well and within what limits the tractor succeeds in performing those functions. How we might measure and represent that performance is also of interest. This output is expressed, as in engineering mechanics, in terms of force (engine torque and drawbar pull), speed (rotational and travel), power (engine and drawbar) and non-dimensional numbers (wheel slip, tractive efficiency). The input is performance is expressed in terms of fuel consumption (actual and per unit power output).

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<sup>1</sup> Hereafter the term 'wheels' will be used to cover all elements unless a specific reference is intended.

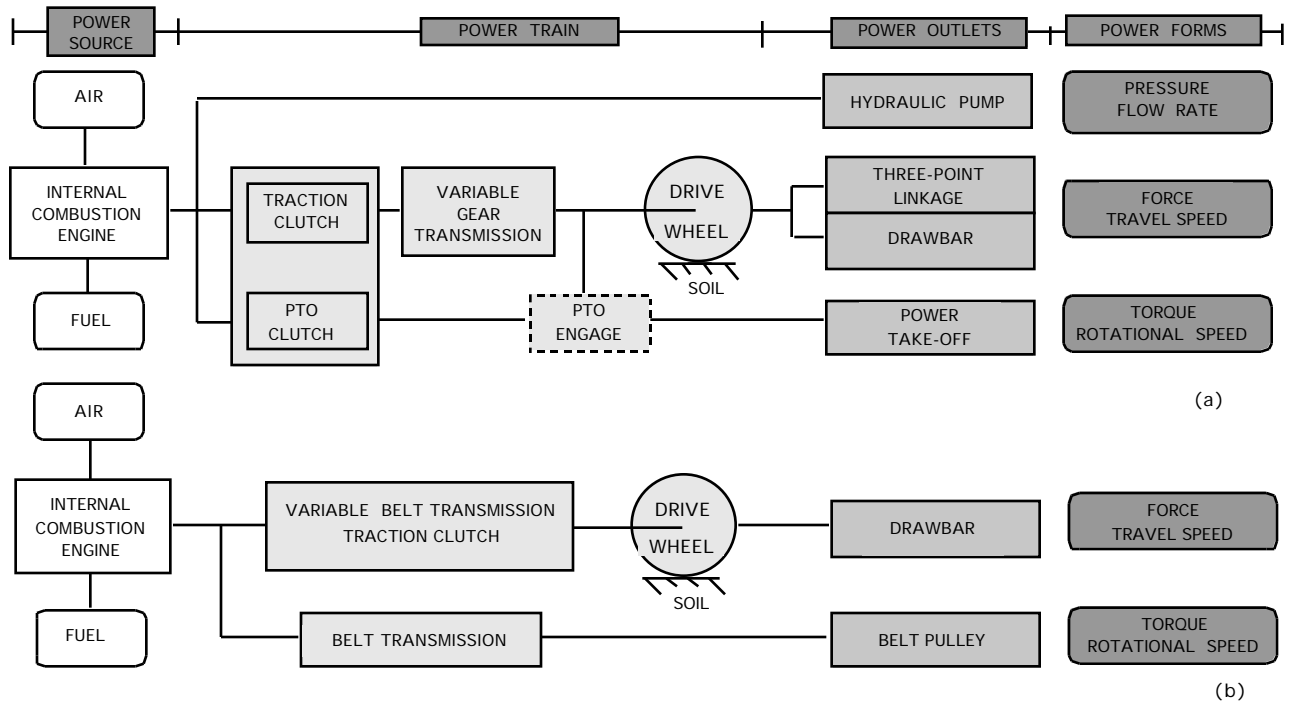


Figure 1.1: Typical power trains (a) for a conventional tractor and (b) for walking tractor / power tiller

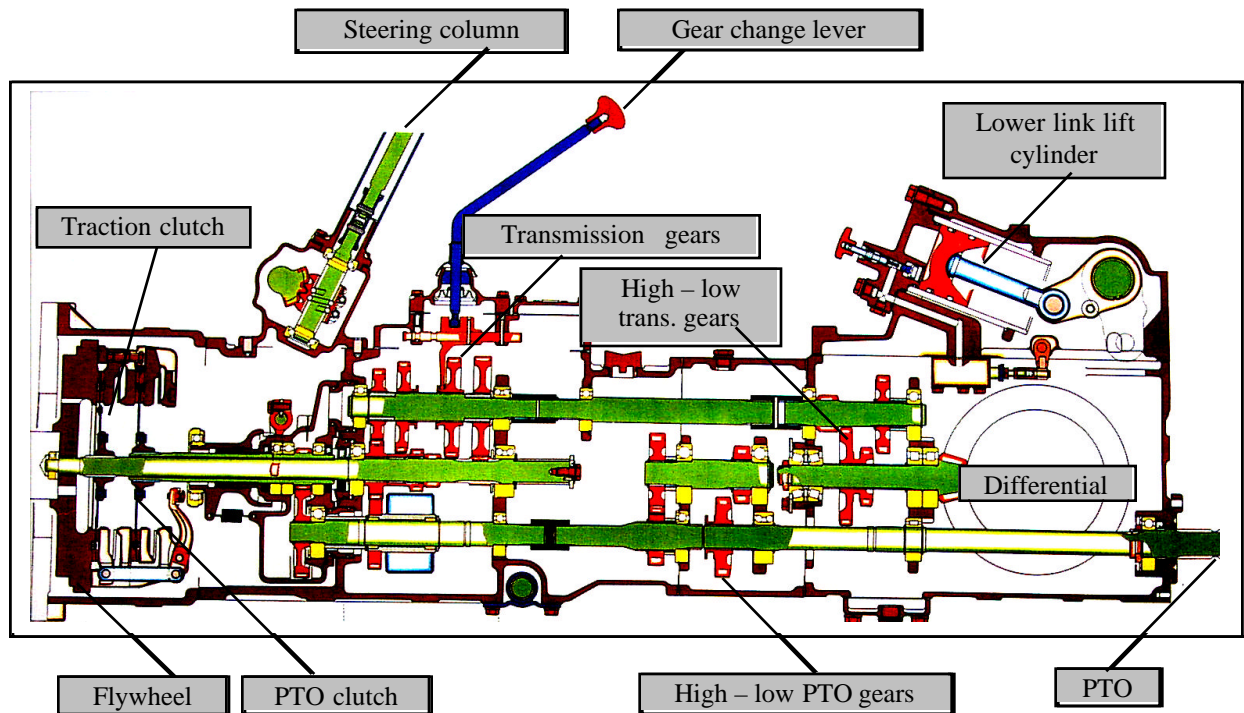


Figure 1.2: Transmission system for a conventional gear drive tractor (Kubota L345)  
Reproduced with permission of Kubota Tractor (Australia)

The overall limitations to performance are also explored in this book as follows:

- (i) At higher travel speeds the limit is engine stall (stopping); optimum engine loading and fuel consumption are achieved by appropriate choice of engine speed and gear ratio.
- (ii) At lower travel speeds in which the limit is wheel slip; the optimum wheel slip is achieved by an appropriate choice of the magnitude of the drawbar load also the weight on and size of the tyres, particularly on the driving wheels.
- (iii) On steep slopes and / or when an incorrect hitch is used; this instability (in the longitudinal plane) is overcome by limiting operation to appropriate slopes and using correct hitching.

Other limitations (not directly associated with performance) such as the actual occurrence of longitudinal and lateral instability, and the loss of steering control due, for example, to vibration, are also beyond the scope of this book.

### 1.3 SYSTEMS AND POWER OUTLETS

Tractors are built in many forms and sizes according to the particular functions that they are required to perform. However, in reviewing their performance it is sufficient to consider the major systems and power outlets that are common to most tractors. The block diagram of the main components in the power transmission system, including the power outlets and forms, is shown in Figure 1.1 (a) for a conventional tractor with PTO and hydraulic power outlets and in Figure 1.1(b) for a walking tractor / power tiller.

The following systems can be identified.

#### 1.3.1 Engine

The engine, which is the immediate source of energy for the operation of the tractor, varies in type and size according to the type and size of the tractor to which it is fitted. It is a mechanism which, using air, extracts the energy from the fuel and transforms it into a mechanical (rotational) form.

Its output (in terms of torque, speed and power) is determined by the physical size of the engine (which determines the amount of air that can be drawn in), the fuel burnt in that air and its speed of operation. Its performance, which is represented in terms of the fundamental characteristic for the engine, ie, the relationship between the torque and (rotational) speed, largely determines and of course limits the performance of the tractor. These are discussed in Chapter 3.

Many other aspects of engine design and operation affect its performance. These include the engine processes (the cycle of strokes on which it operates), the type of fuel and its method of ignition (spark or compression ignition) and the mechanical details such as the design of the components (pistons, crankshaft, valves) and the services such as the lubrication and cooling systems. These details are covered in books on engine design and operation and will not be considered further here.

Engines as used in agricultural tractors may be classified as follows:

- (i) operational cycle      \* two strokes per revolution  
                                     \* four strokes per revolution
- (ii) fuel ignition            \* spark - gasoline, petrol, natural gas  
                                     \* compression - diesel
- (iii) air induction          \* unlimited- diesel  
                                     \* throttled - spark ignition  
                                     \* pressurized - super-charged
- (iv) speed control          \* governed - automatic  
                                     \* ungoverned - manual

### 1.3.2 Power transmission systems and outlets

The transmission systems on the tractor serve to transmit power from the engine to the power outlets, viz:

- (i) traction system (wheels / drawbar / three point linkage)
- (ii) power take off
- (iii) hydraulic (oil) supply

The transmission elements which comprise these systems, may be classified according to their principle of operation:

- (i) mechanical                   \* gears  
                                     \* belts / chains
- (ii) hydrostatic               \* fluid pressure
- (iii) hydro-kinetic           \* fluid momentum   - fluid coupling  
                                                                                 - torque converter

The three transmission systems that transmit power to the three main outlets are discussed below.

#### (a) Traction transmission

##### (i) *Conventional tractors*

The components generally referred to as the 'transmission' and / or the 'gear box' transmit the rotation of the engine to the rear wheels as shown in Figure 1.1 and 1.2. In the conventional tractor this is usually a mechanical system with shafts, gears etc. Only this type will be considered in this book; discussion of the hydro-static system may be found in Goodwin (1979) and of the hydro-kinetic system in Vasey (1957-58).

Because the engine rotates at high speed (a few 1000's of rpm) and the tractor wheels must operate at low speed (a few 10's of rpm), the traction transmission has the function of reducing the speed of rotation of the engine to that required for the rear wheels. Further, because not all operations require the tractor to travel at the same speed, the transmission also has the function of enabling the speed reduction from engine to wheels to be varied by the operator. Thus the travel speed may be changed in from 6 to 12 steps, ie, from about 1 km/hr in a 'low' gear with a 'large' reduction ratio ( $q$  in Chapter 2) to about 20 km/hr in a 'high' gear with a 'small' reduction ratio. The variable ratio is achieved by 'changing gears' (that are in mesh) so that the drive (motion) passes through gears of different sizes (Figure 1.2). This has the effect of altering the overall ratio of the transmission and causing the wheels to run faster or slower.

The (traction) clutch, (Figure 1.2), which is usually of the friction type, is placed between the engine and the transmission. It enables the driver to temporarily disconnect the engine from the rest of the transmission and to make a gradual connection when power transmission is required and the tractor begins to move. Such transmission clutches usually consist of one or more friction surfaces connected to the engine, which are pressed by springs on either side of a disc connected to the remainder of the transmission. Removal of the pressure on the surfaces (disengaging the clutch with the pedal) allows the engine to continue to turn without turning the transmission and the wheels.

That part of the transmission known as the 'differential' has the function of dividing the drive to the wheels and allowing them to turn at different speeds as the tractor turns a corner. Both wheels still drive because the input torques to them remain equal, but they turn at different speeds, corresponding to the respective radii of the curves on which they are travelling. Many tractors have a device to lock the differential. This forces both of the rear wheels to turn at the same speed and so allows the tractor to be driven out of a situation where the differential, in normal operation, allows one wheel to slip and the other to not rotate at all. With the lock engaged the wheel speeds are now equal but the torques are different; hence it is not possible (or difficult) to turn a corner.

A further common component in the transmission is the 'final drive' which consists of speed reduction gears after the differential. These are placed in this position near the wheels to avoid the low speed / high torque in the previous parts of the transmission.

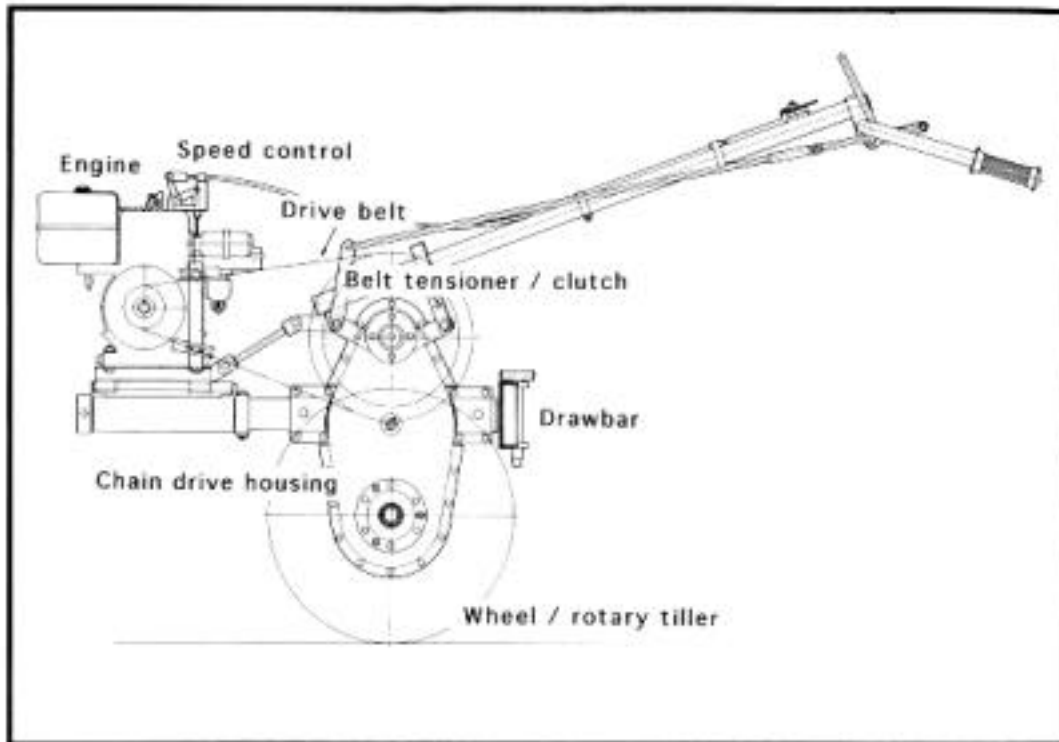


Figure 1.3 (a) Transmission system for walking tractor / power tiller  
 (b) Walking tractor being used for ploughing flooded soil  
 Reproduced with permission of International Rice Research Institute

*(ii) Walking tractor*

In the two-wheel or walking tractor (Figure 1.3), the transmission usually consists of a variable speed V belt drive from the engine, which also acts as a clutch as it is tightened or loosened. A small gear-box may then be fitted, which in turn drives the wheels through chains.

Such tractors are not usually fitted with a power take-off but while stationary may be used to drive equipment such as a pump. The belt drive to the wheels is removed and is used to drive the attached equipment directly.

Power losses in the mechanical transmission systems of tractors are usually small, probably less than 10%.

(b) Power take-off transmission

An ('engine speed') power take-off (PTO) which is frequently fitted to conventional tractors consists of a transmission from the engine to shaft which passes to the outside of the tractor, usually at the rear, and may be engaged to drive attached machines (Figure 1.2). The power passes from the engine through a friction clutch which is frequently operated with the same pedal as the transmission clutch. This, and an engaging mechanism, allows the drive to the power take-off to be stopped and started as required, independently from the drive to the wheels. Hence the driven machine may continue to operate and process the crop even though the tractor and machine are not moving forward. This is a very convenient arrangement and a great advantage over older tractors with a single clutch and especially over ground driven machines.

PTO speed is determined by engine speed, (with a fixed ratio 3 or 4:1) irrespective of travel speed (traction transmission ratio). Power losses in the PTO drive are very small, usually less than 5%.

A "ground-speed" PTO may also be fitted (Fig. 1.1). Here the drive to the PTO shaft is connected to the drive to the wheels after the traction transmission and hence the PTO speed changes as the traction transmission ratio is changed. The ground speed PTO rotates slowly (a few revolutions per unit distance traveled) and may be used as a replacement for a ground drive on machines such as seed drills where a fixed relationship between the movement of the tractor and the function of the machine is important.

The two engaging mechanisms for the PTO drive are such that only one of these can be engaged at one time.

(c) Hydraulic (oil) supply

Here oil under pressure from a hydraulic pump, continuously driven by the engine, is available to operate linear actuators (cylinders, rams) usually for the purpose of controlling (raising and lowering) implements, or driving rotating actuators (motors). One such ram, in-built into the tractor, is used to raise the three-point linkage.

Power losses in the hydraulic system may be moderate but are accepted because this outlet is a flexible and very convenient way of controlling machines and operating auxiliaries on the tractor and on attached machines.

The details of the design and operation of the components in the three tractor transmission systems are covered in books on mechanical analysis and machine design. They will not be considered further in this book.



	Surface	Tread form
(a)	Hard surfaces such as roads	Large area, shallow tread with 'high' pressure
(b)	Normal agricultural work, dry soil	Heavy, intermediate depth tread
(c)	Soft, wet agricultural soils	Deep tread
(d)	Lawns, low sinkage is required	Wide, low pressure
(e)	Dry soil, heavy loads as in earthmoving	Tracks, as on a "crawler" tractor
(f)	Saturated, puddled soils	Metal cage, with angled lugs, alone or as extensions to normal tyres

Figure 1.4 Ground drive elements

(a) to (d) reproduced with permission of Goodyear Tyre Company

(e) reproduced with permission of Caterpillar of Australia, Ltd

(f) reproduced with permission of International Rice Research Institute

### 1.3.3 Wheels

The tractor wheels and associated tyres have the function of supporting the tractor and of converting rotary motion of the engine to linear motion of the tractor as a whole.

The wheels must be chosen to:

- (i) support the weight of the tractor (together with any transferred weight from attached implements) while limiting the sinkage into the soil surface and the resultant rolling resistance.
- (ii) engage with the soil (or surface) and transmit the traction, braking and steering forces (reactions) while limiting relative movement and the resultant slip / skid / side slip.
- (iii) provide ground following ability together with some springing and shock absorption.

The important variables in relation to the tyres include:

- (i) size (diameter and width) which determines their tractive capacity and rolling resistance.
- (ii) strength, expressed in terms of ply rating, which in turn determines the pressure that can be used and hence the weight that the tyre can carry; this in turn also determines the tractive capacity and the rolling resistance.
- (iii) tread pattern which, together with the surface characteristics, determines the engagement and / or contact with the surface.

The losses in power at the wheel / surface interface are often great, particularly on soft surfaces (ie, their efficiency is low), hence the power available at the tractor drawbar may be much less than the power of the engine. Hence the choice of the tyres and the weight on them is crucial in determining the overall performance of the tractor.

Various types of wheels and / or tyres may be used on the tractor, depending mainly on the surface on which it is working. For the following conditions, the tyres or wheels indicated are recommended as shown in Figure 1.4.

## 1.4 STUDYING TRACTOR PERFORMANCE

### 1.4.1 Need for study

Before beginning the study, it may be useful to consider those who have an interest in the subject and why they need to study it.

- (i) The designer wishes to predict whether the tractor being designed will achieve the design objectives. He / she will do this by means of traditional design procedures for mechanical elements such as the power train, experience gained from measurement of the performance of other tractors and the application of the performance prediction techniques explored in this book.
- (ii) Those who are advisers to the users including extension advisers and sales persons also need to understand tractor performance. Their interest is not in design but in how to choose (in economic as well as physical terms) a tractor from a range available to achieve a required work rate (or match other machines) and how to set it up and operate it in the most efficient manner.
- (iii) Users need to understand the basic aspects of tractor performance so that they can interact with their advisers and work their tractors in an efficient manner.
- (iv) Those who are responsible for providing services such as training, administration, safety and other associated aspects to the above groups also need to understand tractor performance and so provide valid and useful advice.

Given their different roles, their need for training material varies widely. This book will not satisfy all groups but may help to provide an understanding of tractor performance and so assist each group in the preparation of associated material needed to fulfil their roles.



### 1.4.2 Approaches to the study

#### (a) Theoretical / ideal

The tractor, which is a machine that is comprised of various simple mechanical elements, can be analysed in terms of their theory. This is presented in Chapter 2 and provides a basic understanding of the operation of the tractor under ideal conditions. However operation of the tractor in the field indicates that this simple analysis is inadequate to determine the limits of its performance as the drawbar load on it is increased, or to predict its performance when operating on soft soils.

#### (b) Practical / experimental

Historically the study of tractor performance has been in practical, experimental terms. In this approach the tractor is operated under described conditions and its performance measured and reported. A similar performance could be expected from another tractor, of the same model when operated under similar conditions, or from a different make of tractor if appropriate allowances were made for any differences, eg, the weight of the tractor or the engine power.

Examination of the results of performance measurements made for tractors operating on soil shows that the condition of the surface is the most significant factor determining their performance. We cannot compare different tractors tested under such conditions because the effects of the inevitable differences in soil condition on the performance are confounded with, and cannot be separated from, the actual differences between the tractors.

Hence, as in other practical measurement approaches, we begin with the performance measured under ideal conditions. This involves testing the engine on a dynamometer and / or the tractor on a hard surface such as a concrete or bitumen road, ie, on a so called 'test track'. Under these conditions we obtain the maximum or best performance that is possible.

Then, if all tractors are tested on the same or similar surface, the surface effect is (at least partly) eliminated. The conclusion from a comparison of such tests then is that tractors ranked in order of some performance parameter (eg, maximum drawbar power or best fuel economy) as obtained on the test track will be the same rank order as if they were tested in actual operating conditions, ie, on a field soil. This is the same logic as used when we measure the strength of various steels in a testing machine and hence rank the strength of beams made from them.

The reports of formal tractor testing schemes (Nebraska, OECD, etc) and many other research papers are examples of the practical / experimental approach.

Tractor performance as measured in this way is described in Chapter 3 and is satisfactory as far as it goes. However it does not provide a fundamental understanding of the traction process, nor does it provide a basis for the prediction of performance which is the basis of engineering design.

#### (c) Theoretical / predictive

In this approach we set up a theoretical model (based, like all theoretical work, on some empirical or experimental data) of the way in which the wheels interact with the soil:

- (i) in the vertical direction as it supports the vehicle.
- (ii) in the horizontal direction as it generates the reaction to provide the drawbar pull.

The early work by Bekker (1956) and later work by Reece (1965-66 and 1967) and many others uses the standard properties of the soil (cohesion and angle of internal friction) and an empirical deformation parameter to characterize its strength and deformation properties respectively. These are used to model the generation of shearing stresses within the contact area which are then integrated to give the total reaction of the soil and hence the drawbar pull and power. This is presented in Chapter 4, Sections 4.3 to 4.6.

This approach provides a good understanding of the traction processes and of the effect of the dimensional characteristics of the wheel and the strength properties of the soil. However its application for field use is limited because it involves the complex and time consuming, in-situ measurement of the three soil properties.

(d) Empirical / predictive

This approach is predictive but is based entirely on empirical relationships that have been established between a single soil parameter (together with the dimensions of the wheel) and the tractor performance (Wismer and Luth (1974) . The easily measured parameter (cone index), represented by the force to push a cone into the soil divided by the cross sectional area of the cone, is a complex but ill-defined measure of soil strength and compressibility.

This is a rapid and versatile method of predicting the field performance of tractors. However again it does not provide a basic understanding of the traction process but it does allow a rapid representation of the overall performance as shown in Chapter 5.

**1.5 PREVIEW**

The theory and explanation which follows in the later Chapters applies to the conventional rear-wheel drive tractor irrespective of what form other features, such as the engine, transmission or steering, may take. With appropriate modifications, as noted in the text, it may also apply to other forms such as the crawler and walking tractors.

In general it does not apply to the four-wheel drive type because with such a system, the drive is divided in an unknown proportion between the front and rear axles in a way that depends on the stiffness of the respective drive trains to the wheels. It also depends on the strength and stiffness of the soil in the soil / wheel contact patch which in turn depends on the respective weights on these wheels.

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## **APPENDICIES**

### **I LIST OF SYMBOLS**

### **II DIMENSIONAL DATA FOR FARMLAND TRACTOR**

# APPENDIX I

## LIST OF SYMBOLS

Symbol	Definition	Defining section
a	distance from drawbar to implement wheels parallel to ground surface	6.4.3
a	constant in normal stress distribution characteristic	4.7
b	distance from drawbar to soil force on implement parallel to ground surface	6.4.3
b	distance from drawbar to trailer wheels	6.4.4
b	width of plate, tyre	4.3.2
c	cohesion of soil	4.4.2
d	depth of cultivation	7.5
d	tyre diameter	5.3.1
d'	constant in implement draught - speed characteristic	7.2.3
h	tyre section height	5.3.1
h	ratio drawbar height / centre of gravity height	6.5
i	wheelslip	2.3.1
i'	wheelslip at maximum tractive power	4.6.2
j	deformation of soil parallel to soil surface	4.4.2
k	shear deformation modulus of soil	4.4.2
k	rate constant	5.3.2
$k_c, k$	sinkage moduli of soil	4.3.2
$\ell$	length of plate, distance traveled by wheel	4.3.2
$\ell$	length wheel / track contact with ground	4.3.2
m	distance travelled by wheel	3.3.3
n	number of revolutions of wheel	4.1.5
n	sinkage exponent of soil	4.3.2
p	plate, wheel pressure on soil	4.3.2
q	transmission ratio	2.2.1
r	wheel radius	6.3.1
t	time period	3.2.3
w	width of implement	7.5
x	distance along track	4.4.4
x	distance between the two axles, parallel to the ground surface (wheel base)	6.3.1
x'	distance from rear axle to hitch point (or point of application of implement load) parallel to ground surface	6.4.1
x''	distance between the two axles parallel to the ground surface (tractor raised)	6.3.1
$x_f$	distance from front axle to centre of gravity of tractor	6.3.1
$x_r$	distance from rear axle to centre of gravity of tractor	6.3.1
$x_h$	distance from axle to the handle (walking tractor) parallel to the ground surface	6.4.4
$x'_r$	horizontal distance from rear axle to weight (tractor raised)	6.3.1
y	distance from rear axle to hitch point (or point of application of implement load), perpendicular to the ground surface	6.4.1
y'	distance from ground contact point to hitch point (or point of application of implement load), perpendicular to the ground surface	6.4.1
y''	height of front axle (tractor raised)	6.3.1
$y_g$	distance from rear axle to centre of gravity for tractor perpendicular to ground surface	6.3.1
$y_t$	distance from axle to centre of gravity for trailer perpendicular to ground surface	6.4.4
z	deformation (sinkage) of soil perpendicular to the soil surface	4.3.2
z	distance from ground surface to point of application of soil force perpendicular to the ground surface	6.4.3
z'	slope distance from rear axle to weight (tractor raised)	6.3.1

A	area of wheel, track contact with ground	4.4.3
A	constant	5.2
B	constant	5.2
C	calorific value of fuel	2.4.1
CI	cone index	5.3.1
D	wheel diameter / draught	2.2.1
D	implement draught	7.2.2
F	lift force on implement drawbar perpendicular to the ground surface	6.4.4
FC	fuel consumption rate	3.2.3
H	tractive force / soil reaction parallel to the ground surface	2.2.2
M	mobility number	5.3.1
M	moment on wheel / chassis	6.4.1
N	rotational speed	2.2.1
N	number of revolutions	4.1.5
P	drawbar pull, weight of attached implement	2.2.1
Q	power	2.2.3
Q'	tractive power	4.6.2
R	rolling resistance	2.3.2
R	weight on trailer wheels	6.4.4
S	shear stress	4.4.2
S	soil force	6.4.3
SFC	specific fuel consumption	3.2.1
T	torque	2.2.2
T	force on implement at tractor drawbar, perpendicular to the ground surface	6.4.3
U	force on handles, perpendicular to slope	6.4.4
V	travel speed	2.2.1
V	dynamic weight on wheels	6.4.1
V <sub>s</sub>	slip velocity of wheel relative to surface	2.3.3
V'	dynamic weight on implement wheels	6.4.4
W	weight of tractor	6.3.1
W	static weight on wheels	6.3.1
W'	weight of trailed implement, trailer	6.4.1
W' <sub>f</sub>	weight on front wheels (tractor raised)	6.3.1
X	slip function	4.4.4
	angle of slope of ground surface	6.4.1
	angle	6.3.1
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### Subscripts

d	drawbar / down
e	engine
f	front wheel
g	centre of gravity
h	handles
n	transmission
o	theoretical, ideal, zero load, overall, zero speed
r	rear wheel
s	static, slip
t	trailer, traction
u	up

## APPENDIX II

DIMENSIONAL DATA FOR FARMLAND TRACTOR			
FEATURE	Symbol	Values for	
		Farmland tractor	Local tractor
<b>Weights</b>		<b>kg / kN</b>	<b>kg / kN</b>
Total weight	W	2850 / 27.9	
Weight on rear wheels (on horizontal ground)	$W_r$	2030 / 19.9	
Weight on front wheels (on horizontal ground)	$W_f$	820 / 8.0	
<b>Dimensions</b>		<b>metre</b>	<b>metre</b>
Wheel base (front to rear axle)	x	1.88	
Rear axle to C of G (parallel to ground)	$x_r$	0.54	
Front axle to C of G (parallel to ground)	$x_f$	1.34	
Rear axle to C of G (perpendicular to ground)	$y_g$	0.13	
Ground contact to drawbar (parallel to ground)	$x'$	0.60	
Ground contact to drawbar (perpendicular to ground)	$y'$	0.45	
Rear axle to drawbar (perpendicular to ground)	y	0.185	
Rear wheel rolling radius (on 14.9 x 28 tyres)	r	0.635	
<b>Overall transmission ratio</b>	q	<b>Ratio</b>	
Gear 1		221.1	
Gear 2		170.0	
Gear 3		139.5	
Gear 4		108.9	
Gear 5		85.6	
Gear 6		67.4	
Gear 7		47.3	
Gear 8		37.1	
Gear 9		22.0	
Gear 10		17.3	
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# **The Mechanics of Tractor – Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

## **CHAPTER 2 TRACTOR MECHANICS**

***Printed from: <http://www.eprints.unimelb.edu.au>***

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 2

### TRACTOR MECHANICS

#### 2.1 INTRODUCTION

The tractor is a machine and the application of the general principles of mechanics to it provides a simple but fundamental understanding of its operation and ideal performance. The actual performance will be less than this, and may be much less, mainly because of the losses which occur at the wheel / ground contact surface.

In a similar way to other engineering disciplines, we can define the elements or components of the tractor in terms of general mechanics without needing to know their detailed form. Thus the engine (power source) can be represented in terms of its torque and speed without having to specify its type (thermodynamic or electrical), its operating principle (internal or external combustion), its operating cycle (two or four stroke) or its fuel source (diesel or petrol (gasoline)). Similarly the transmission system can be expressed in terms of the transmission ratio without specifying its form or operating principle (mechanical (gears, chains, belts), hydrostatic (fluid pressure) etc).

We can thus separate the application of the principles of mechanics to the tractor from the particular forms of the mechanisms that appear in the particular tractor that we see in the laboratory or field.

#### 2.2 IDEAL ANALYSIS (without losses)

Consider a tractor operating on a firm surface as shown in Figure 2.1. Although the tractor is moving, the equations of equilibrium can be applied to it because it is assumed that there is no acceleration.

Consider the engine running at a rotational speed  $N_e$  driving the drive wheels without losses through a transmission with an overall ratio of  $q$ . As a consequence of the reduction in speed by a factor of  $1/q$ , there is a corresponding increase in torque by a factor of  $q$ . These values correspond to the 'velocity ratio' and the 'mechanical advantage' from elementary physics.

##### 2.2.1 Speed analysis

For the tractor as shown in Figure 2.1(a):

Drive wheel diameter =  $D$

Engine speed =  $N_e$

Overall transmission ratio  $q = \frac{\text{Engine speed } N_e}{\text{Drive wheel speed } N_w}$

Drive wheel rotational speed  $N_w = \frac{N_e}{q}$

If we assume that there are no losses in motion due to slip between the wheel and the surface:

$$\begin{aligned} \text{Travel speed, } V_o &= \text{Linear speed of wheels} \\ &= D N_w \\ &= \frac{D N_e}{q} \end{aligned} \quad (2.1)$$

This analysis shows that the travel speed depends directly on the engine speed and inversely on the gear ratio.

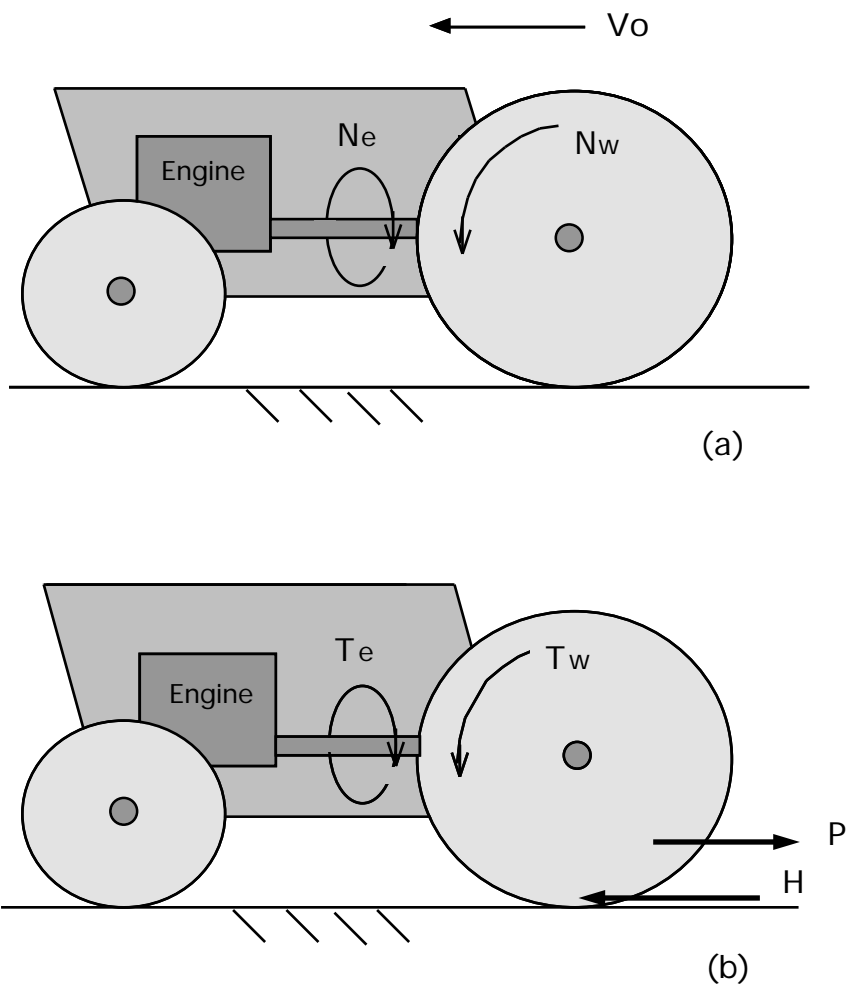


Figure 2.1 Mechanics of the tractor under ideal conditions  
 (a) Speed analysis; (b) Torque / force analysis

### 2.2.2 Torque / force analysis

For the tractor as shown in Figure 2.1(b):

$$\text{Engine torque} = T_e$$

$$\text{Drive wheel torque, } T_w = q T_e$$

Equilibrium requires that this torque is equal and opposite to the moment of the soil reaction,  $H$  on the wheel:

$$H \frac{D}{2} = T_w = q T_e$$

$$H = \frac{2 q T_e}{D}$$

If we assume that there are no other horizontal external forces acting (such as rolling resistance), equilibrium also requires that:

Drawbar pull,  $P = \text{Soil reaction, } H$

$$P = \frac{2 q T_e}{D} \quad (2.2)$$

This analysis shows that the drawbar pull depends directly on the torque generated by the engine and on the gear ratio. This assumes that the wheel / ground contact can generate the reaction to  $P$ .

### 2.2.3 Power analysis

$$\text{Engine power, } Q_e = 2 T_e N_e \quad (2.3)$$

$$\text{Drawbar power, } Q_d = \text{Drawbar pull} \cdot \text{travel speed}$$

$$= P \cdot V_o \quad (2.4)$$

$$= \frac{2 q T_e}{D} \cdot \frac{D N_e}{q}$$

$$= 2 T_e N_e$$

$$= \text{Engine power}$$

Thus, if we neglect losses in forward motion due to wheelslip and in drawbar pull due to rolling resistance, all of the power from the engine is available at the drawbar.

The above represents the ideal situation which might apply approximately to the tractor working on hard surfaces with small drawbar pulls and small wheelslips.

However, in many agricultural situations, wheelslip is significant, hence the travel speed of the tractor will be less, and may be much less, than the ideal value calculated above. Also, much of the torque on the rear wheels goes to drive the tractor forward against the rolling resistance of both the driving and the rolling wheels. Hence the drawbar pull will be less, and may be much less, than the ideal value calculated above.

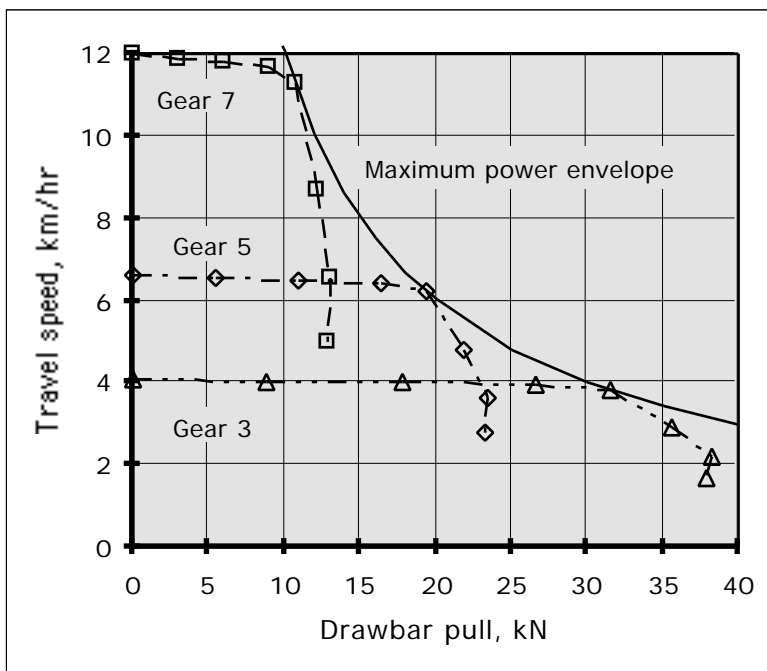
The actual tractive performance of the tractor in various gears on two types of surface, viz., a hard surface (firm, dry soil or road) and a soft surface (cultivated soil), is considered in Chapters 3 and 4, respectively.

Engine		Ideal tractor drawbar performance in gears					
Speed	Torque	Drawbar Pull, kN			Travel speed, km/hr		
rpm	Nm	Gear 3	Gear 5	Gear 7	Gear 3	Gear 5	Gear 7
Gear ratio->		139.5	85.6	47.3	139.5	85.6	47.3
2390	0	0.00	0.00	0.00	4.07	6.63	11.99
2370	40	8.86	5.43	3.00	4.03	6.57	11.89
2350	80	17.71	10.87	6.01	4.00	6.52	11.79
2325	120	26.57	16.30	9.01	3.96	6.45	11.67
2250	142	31.51	19.33	10.68	3.83	6.24	11.29
1730	161	35.65	21.88	12.09	2.94	4.80	8.68
1300	173	38.31	23.51	12.99	2.21	3.61	6.52
1000	171	37.86	23.23	12.84	1.70	2.77	5.02

(a)

Maximum power performance envelope	
Drawbar Pull	Travel Speed
kN	km/hr
6.0	20.10
7.0	17.23
8.0	15.08
10.0	12.06
12.0	10.05
14.0	8.61
16.0	7.54
18.0	6.70
20.0	6.03
25.0	4.82
30.0	4.02
35.0	3.45
40.0	3.02

(b)



(c)

Figure 2.2: Data for (a) ideal performance of Farmland tractor in 3 gears at maximum governor setting; (b) maximum power envelope; (c) plot of these data

**Problem 2.1**

For a local tractor (of any type):

- (a) Measure the transmission ratios in each gear by (securely) raising the drive wheels and either:
  - (i) turning the engine by hand and counting revolutions of engine and wheels
  - (ii) running the engine and measuring the speed of engine and drive wheels with a tachometer
- (b) Check your answers by:
  - (i) taking appropriate measurements of the transmission elements - counting gear teeth, measuring pulley or sprocket diameters etc
  - (ii) driving the tractor on a hard surface and measuring the travel speed, and rolling radius
  - (iii) inspection of the owner's manual or parts book, if available.

**2.2.4 Ideal performance graphs**

Figure 2.2 shows the torque (Nm) - engine speed (rpm) data from an actual test on the engine from the hypothetical 'Farmland' tractor <sup>1</sup>. It also shows the ideal performance (travel speed (km/hr) versus drawbar pull (kN)) graphs for the Farmland tractor in 3 gears based on the Equations 2.1 and 2.2 and data from Table 1, Appendix I.

The shape of these graphs will be discussed more fully in Chapter 3.

**Problem 2.2**

Plot similar graphs for the other gears of the Farmland tractor.

**2.2.5 Performance envelopes**

The graphs shown in Figure 2.2 and others to be plotted in Problem 2.2 give the characteristic graphs for the tractor with discrete gears. Such gears result in 'steps' in the curves defining areas in which the tractor can work and other areas between the steps in which the engine could work but which are unavailable because gears with appropriate ratios are not fitted to the tractor.

If the tractor were fitted with a stepless or infinitely variable transmission, the ratio could be varied to keep the engine operating at maximum power. This would give the (ideal) performance 'envelope' or boundary within which the tractor must work. This is also shown in Figure 2.2 (c) for the constant maximum power of the engine (33.6kW); it is plotted for arbitrarily chosen values of the drawbar pull and calculated travel speeds shown in Figure 2.2(b).

**2.2.6 Conclusion**

The simple analysis given above suggests that the actual performance of the tractor will reflect the performance of the engine:

- (i) travel speed is determined by engine speed
- (ii) drawbar pull determines engine torque
- (iii) both travel speed and torque also depend on transmission ratio.

Further, the travel speed - drawbar pull performance is limited by the maximum engine power envelope which appears as an hyperbola on the travel speed / drawbar pull graph space.

As shown later in Chapter 3, the actual travel speed - drawbar pull graphs and the corresponding envelope will be different because losses in travel speed due to wheelslip, in drawbar pull due to rolling resistance and in power due to both.

<sup>1</sup> Test data have been extracted from Australian Tractor Test Report No 78 (Brown and Baillie, 1973). Other numerical data for this tractor, which are used in this book, have been extracted and are presented in Appendix II.

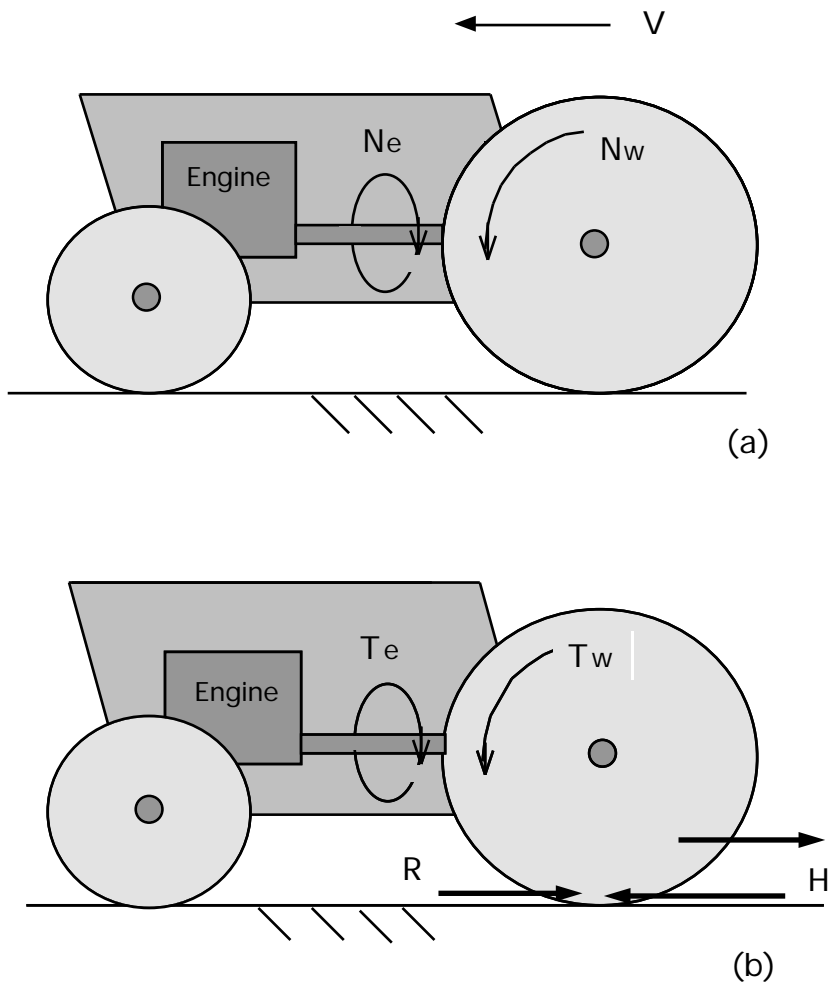


Figure 2.3 Mechanics of the tractor with losses  
 (a) Speed analysis; (b) Torque / force analysis



### 2.3 ANALYSIS WITH LOSSES

Consider a tractor again operating on a firm surface as shown in Figure 2.3. Although the tractor is again moving, the equations of equilibrium can be applied to it because it is assumed that there is no acceleration.

#### 2.3.1 Speed analysis

The tractor is now moving with a speed  $V$  (less than the ideal travel speed,  $V_o$  above), Figure 2.3(a).

We can then define wheelslip as:

$$\text{Wheelslip, } i = \frac{V_o - V}{V_o} \quad (2.5)$$

Where,  $V_o$  = theoretical travel speed (as in Equation 2.1 above)

$V$  = actual travel speed

Substituting for  $V_o$  from Equation 2.1

$$V = V_o (1 - i) = \frac{D N_e}{q} (1 - i) \quad (2.6)$$

#### 2.3.2 Force analysis

A rolling resistance force ( $R$ ) which is assumed to act horizontally on the wheel at the wheel / ground contact patch, opposes motion of the tractor, Figure 2.3(b).

For equilibrium of the external horizontal forces acting on the tractor:

$$H = P + R \quad (2.7)$$

#### 2.3.3 Power analysis

Considering power transmission at the wheels.

$$\begin{aligned} \text{Output power} &= \text{Input power} - \text{Power loss} \\ \text{ie, Drawbar power} &= \text{Wheel power} - \text{Power loss} \\ \text{Hence, Power loss} &= \text{Wheel power} - \text{Drawbar power} \\ &= 2 T_w N_w - P V \\ &= 2 H \frac{D}{2} \frac{V_o}{D} - P V = H V_o - P V \\ &= H V_o - (H - R) V = H (V_o - V) + R V \\ &= H V_o i + R V = H V_s + R V \end{aligned} \quad (2.8)$$

Here  $V_s$  is the slip velocity, ie, the velocity of the wheel relative to the surface at the surface / wheel contact.

We can identify the terms in this equation as:

$$\text{Total power loss} = \text{Power loss due to slip} + \text{Power loss due to rolling resistance}$$

Minimizing the total power loss thus is matter of minimizing the sum of the loss due to slip and that due to rolling resistance. This is a complex problem when it is realized, for example, that the effect of weight on the driving wheels is to decrease the slip loss and increase rolling resistance loss. This will be discussed further in Chapter 4.

## 2.4 OTHER MEASURES OF PERFORMANCE

### 2.4.1 Efficiency

#### (a) Tractive efficiency

We define tractive efficiency,

$$\begin{aligned} \eta_t &= \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Drawbar power}}{\text{Wheel power}} \\ &= \frac{P \cdot V}{H \cdot V_o} = \frac{(H - R)}{H} (1 - i) \end{aligned} \quad (2.9)$$

$$= \left(1 - \frac{R}{H}\right) (1 - i)$$

$$= \frac{P}{(P + R)} (1 - i) \quad (2.10)$$

The tractive efficiency that appears here contains two terms:

- (i)  $\frac{P}{(P+R)}$  which represents a 'force' efficiency; thus when there is no rolling resistance ( $R = 0$ ) this factor in the tractive efficiency = 1.
- (ii)  $(1 - i)$  which represents a 'speed' efficiency; again when there is no wheelslip ( $i = 0$ ), this factor in the tractive efficiency = 1.

It might be thought that the tractive efficiency, which is one of the most important measures of tractor performance, could be determined on the basis of Equation 2.10. However, the major difficulty with this approach is that, in practice, it is not possible to determine a relationship between rolling resistance and slip or, in general, to determine rolling resistance when a wheel is undergoing a slip.

Hence, it is necessary to determine the tractive efficiency by measuring drawbar and wheel power directly by measuring:

- (i) drawbar pull,  $P$ , with a tension load (force) cell between the tractor and a load vehicle or implement
- (ii) travel speed,  $V$ , by timing over a known distance
- (iii) wheel torque,  $T_w$ , with a torque load cell in the transmission to the driving wheels
- (iv) wheel speed,  $N_w$ , by counting wheel revolutions over a known time period

$$\text{Then tractive efficiency, } \eta_t = \frac{P V}{2 T_w N_w} \quad (2.11)$$

#### (b) Transmission efficiency

We can define transmission efficiency:

$$\eta_r = \frac{\text{Power to wheels}}{\text{Power from engine}} = \frac{2 T_w N_w}{2 T_e N_e} \quad (2.12)$$

The maximum transmission efficiency is dependent on the design and the quality of the transmission elements. In a geared transmission there is little or no loss in velocity,  $N_w = N_e / q$ .

Hence any losses are due to a loss in torque; thus  $T_w < q \cdot T_e$

For good quality gears the maximum efficiency is about 98% per pair of gears; hence with, say, 3 pairs of gears in the change transmission and another 2 pairs in the differential / final drive, the maximum efficiency will be  $(0.98)^5 = 90\%$ . Little improvement in efficiency can be obtained by more accurate or elaborate gearing; other types of transmission will be no more efficient.

(c) Engine efficiency

We can define engine efficiency:

$$= \frac{\text{Power from engine}}{\text{Power in fuel}} = \frac{2 T_e N_e}{1000 FC C} \quad (2.13)$$

where  $FC$  = fuel consumption rate, kg/min  
 $C$  = calorific value of the fuel, kJ/kg

The maximum value for engine efficiency is dependent on and strictly limited by the thermodynamics of the engine processes. A maximum value of about 35% for a diesel engine can be expected; other types of engine will, in general, be less efficient.

(d) Overall efficiency

We can also define the overall efficiency for the tractor:

$$\begin{aligned} &= \frac{\text{Drawbar power}}{\text{Fuel power}} \\ &= \frac{\text{Engine power}}{\text{Fuel power}} \cdot \frac{\text{Wheel power}}{\text{Engine power}} \cdot \frac{\text{Drawbar power}}{\text{Wheel power}} \\ &= \text{Engine efficiency} \cdot \text{Transmission efficiency} \cdot \text{Tractive efficiency} \\ &= \quad \cdot \quad \cdot \end{aligned} \quad (2.14)$$

Consider typical maximum values for these variables:

$$\begin{aligned} &= 0.3 \times 0.90 \times 0.75 \\ &= 20 \% \end{aligned}$$

Because the maximum tractive efficiency is low and highly variable and the other efficiencies are high (transmission) or strictly limited (engine), any significant increase in the overall efficiency of tractor performance will be achieved by increasing the tractive efficiency. Research into an understanding of the traction process and into more efficient traction devices is directed to this end.

### 2.4.2 Tractive coefficient (pull - weight ratio)

As will be shown later, the performance of a tractor depends to a significant degree on its weight and, in particular, on the weight on the driving wheels. It is therefore useful to define a non-dimensional drawbar pull - weight ratio termed:

$$\text{Tractive coefficient, } = \frac{\text{Drawbar pull}}{\text{Weight on driving wheels}} \quad (2.15)$$

The tractive coefficient is a number which characterizes the interaction between the wheel and the surface in an analogous way to which coefficient of (sliding) friction characterizes the interaction between one body sliding on another. Where a different wheel and surface may be considered similar to those for which the tractive coefficient is known, then for the same wheelslip:

$$\text{Drawbar pull} = \text{Tractive coefficient} \times \text{weight on wheel}$$

Where a tractor operates on a slope the tractive coefficient should logically be based on the total force parallel to the ground, ie, on the drawbar pull plus the component of the weight of the tractor down the slope.

Where a four-wheel tractor is considered, and with other tractors also, the weight used may be the total weight on all wheels. In quoting values of tractive coefficient, it is therefore necessary to state which weight has been used.

#### Problem 2.3

Estimate the maximum pull - (total) weight ratio for some local traction devices, eg, tractor, locomotive, draught animal or human.

## 2.5 SUMMARY

### TRACTOR PERFORMANCE PARAMETERS

Parameter	Engine	Transmission	Wheels
<b>Input force</b> * Force conversion ratio * Theoretical force * Force losses <b>Output force</b>	Combustion pressure Variable with rotation - Mechanical friction Engine torque, $T_e$	Engine torque, $T_e$ Gear ratio, $q$ Engine torque $\times$ gear ratio Mechanical friction Wheel torque, $T_w$	Wheel torque, $T_w$ Force radius Tractive force, $H$ Rolling resistance, $R$ Drawbar pull, $P$
<b>Input velocity</b> * Velocity conversion ratio * Theoretical output velocity * Velocity losses <b>Output velocity</b>	Piston velocity Variable with rotation Engine speed, $N_e$ Nil Engine speed, $N_e$	Engine speed, $N_e$ Gear ratio, $q$ Engine speed / gear ratio Nil Wheel speed, $N_w$	Wheel speed, $N_w$ Rolling radius Wheel linear speed, $V_o$ Wheelslip, $i$ Travel speed, $V$
<b>Input power</b> * Theoretical output power <b>Output power</b>	Fuel power - Engine power, $Q_e$	Engine power, $Q_e$ - Wheel power, $Q_w$	Wheel power, $Q_w$ Tractive power, $Q_t$ Drawbar power, $Q_d$
<b>Input/output efficiency</b>	Fuel efficiency, $f$	Transmission efficiency, $r$	Tractive efficiency, $t$

Table 2.1 Summary of tractor performance parameters (Parkhill, Pers. comm)

## 2.6 REFERENCES

Brown, W.T. and Baillie, W.F. (1973) *Australian Tractor Test Report No 78, Leyland 253*. Australian Tractor Testing Committee, University of Melbourne.

Parkhill, J. G., Personal communication

# **The Mechanics of Tractor - Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

## **CHAPTER 3**

### **TRACTOR PERFORMANCE ON FIRM SURFACE**

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 3

### TRACTOR PERFORMANCE ON A FIRM SURFACE

#### 3.1 INTRODUCTION

We begin the study of tractor performance in detail by considering the performance of a conventional two-wheel drive tractor when operating on a firm surface.

As shown in Chapter 2 the ideal performance of a tractor reflects the performance of the engine and the transmission.

- (i) The travel speed depends directly on the engine speed, inversely on the transmission ratio and, when speed losses are considered, on the wheelslip.
- (ii) The drawbar pull depends directly on the engine torque, on the transmission ratio and, when force losses are considered, on the rolling resistance.
- (iii) The drawbar power directly on the engine power and the losses through the transmission and at the wheel / ground surface as in (i) and (ii) above.

The actual performance of tractors has traditionally been determined by measurement during practical / experimental tests of their engines and the complete tractor operating under controlled and repeatable conditions as discussed in Section 1.4.2 (b) above.

In Chapter 3 we consider a conventional rear wheel drive tractor driven by a diesel engine through a transmission with discrete gears. The tractor was set up with tyres (size and weight) and other conditions as recommended by the manufacturer. It was then operated to explore the two variables that are open to choice by the operator, viz, governor setting and gear selected.

The testing is done:

- (i) with the engine driving a rotary dynamometer or brake. Here the speed of the engine varies with the torque load on it for various settings of the governor as determined by the operator. The fuel consumption and efficiency of the engine are also measures of its performance.
- (ii) with the tractor being operated on a firm surface. Here the travel speed varies as the drawbar load is varied. The transmission ratio (the gear), as selected by the operator, influences the performance because it determines the condition under which the draught load is matched to the output of the engine. The efficiency of the transmission which is high and nearly constant is not a significant variable.

The example given is for the hypothetical 'Farmland' tractor based on a selection of results from an Australian Tractor Test Report No 78 (Brown and Baillie, 1973). Other data which are used in this book, have been extracted and are presented in Table 1, Appendix II.

The performance of the tractor is presented in graphical form. A detailed discussion of this technique is presented in Vasey and Baillie (1969).

The following discussion is generally applicable to tractors with governed diesel engines (since these are now most commonly used) although most of the principles would apply to the performance of tractors with other forms of engine. Also, while the discussion is given mainly in terms of a four-wheel tractor, the same principles would generally apply to a two-wheel tractor (Pudjiono and Macmillan, 1995).

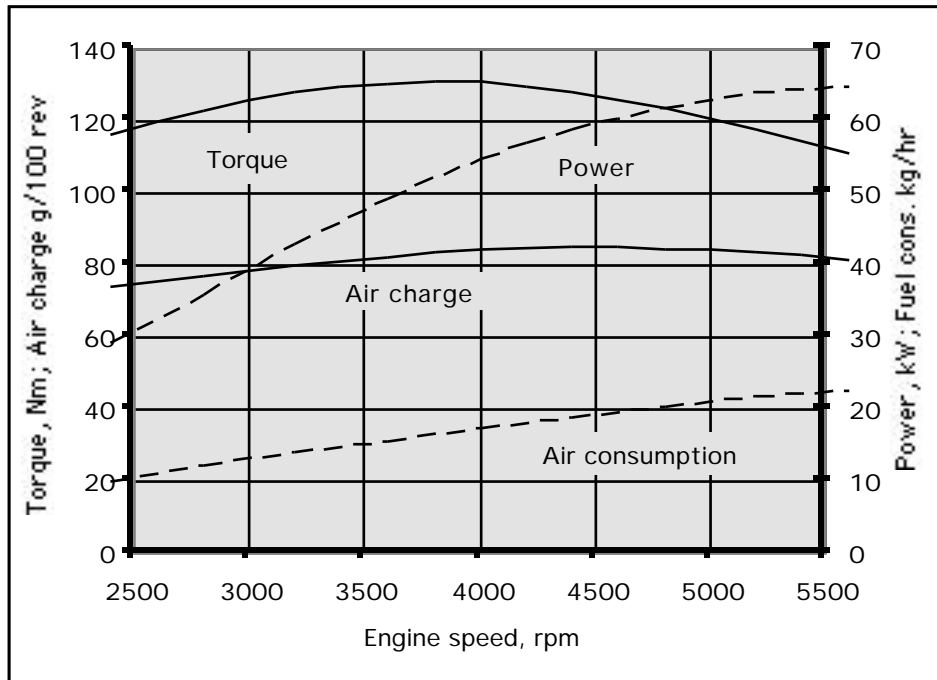


Figure 3.1: Variation in air charge and torque also air consumption rate and power with engine speed. Reproduced from data in Goulburn and Brown (1993) with permission by Mechanical Engineering Publications / Professional Engineering Publishing Ltd.

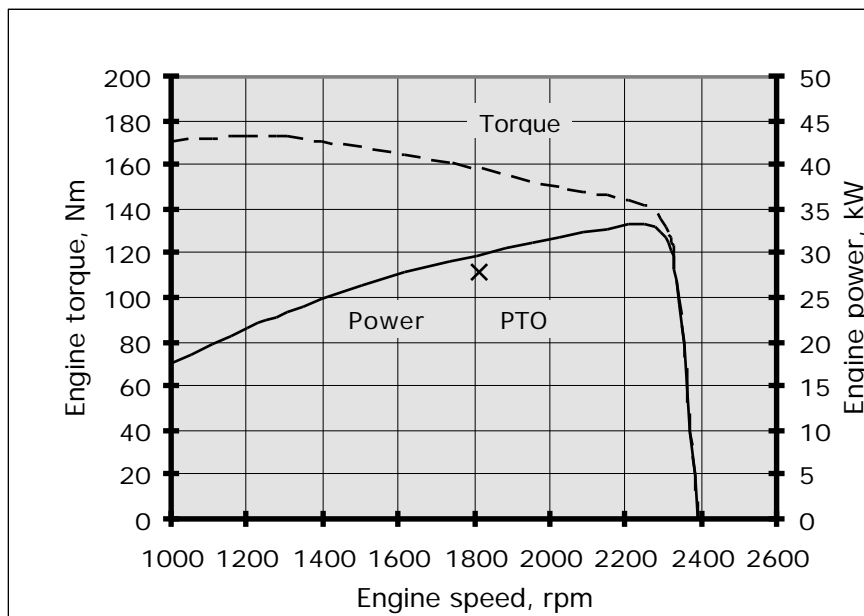


Figure 3.2: Variation of engine torque and power with speed for the Farmland tractor engine at maximum governor setting; data from Figure 2.2.

### 3.2 ENGINE PERFORMANCE

#### 3.2.1 General

The detailed operation and performance of the diesel engine is presented in many text books, hence the discussion here will be limited to its input and output performance characteristics.

##### (a) Output

This is transmitted from the crankshaft in a rotational form, hence it is measured in terms of:

- (i) torque - rotational effort, Nm
- (ii) speed - rotational motion, rad/sec or rpm

The output will be represented by the way in which the torque developed by the engine (equals torque load applied to the engine) varies with its (rotational) speed.

##### (b) Input

This is in the form of:

- (i) air drawn into the engine acting as a pump (air charge)
- (ii) fuel metered into the air:
  - \* already in the cylinders for diesel engine
  - \* by the carburetor during its passage to cylinders for a spark ignition engine

The maximum output of the engine is effectively determined by the maximum input, the limiting factor being the quantity of air (charge) drawn into the cylinder on each stroke (Goulburn and Brown, 1993). This in turn will depend on:

- (i) the size of the cylinders
- (ii) the restriction offered by the air passages, valves, etc
- (iii) the time available for the air to be drawn in

For a given engine:

- (i) at high speed, the time available for the air to enter the cylinders is so short that the air charge is reduced;
- (ii) at low speed, the time available for the air to enter the cylinders is longer but heating of the air in the cylinder reduces the charge

Hence, for a given engine, there is an optimum speed at which most air is drawn in; at both higher and lower speeds, less air enters (Figure 3.1).

Because the output (torque) from the engine depends on input (air), the maximum output (torque) coincides approximately with maximum air charge. Strictly, this statement is only true for a fixed air / fuel ratio, as determined by the amount of fuel which can be effectively burnt in the air available. More fuel will give slightly greater output torque, but most of the extra fuel will be wasted and will appear as black, un-burnt carbon in the exhaust gas.

#### 3.2.2 Output

##### (a) Torque - speed

The torque output represents the magnitude of the rotational effort developed by the engine against a torque load applied to it. The torque-speed graph for an un-governed engine shows a very wide range of speed as the torque load is varied; see Figure 3.1.

In operation the load on a tractor and hence the torque on the engine varies widely and in an unpredictable way, which would cause the tractor to slow down and speed up according to the load. This would be unsuitable, particularly for many PTO driven machines such as cereal harvesters or forage mowers where a constant PTO speed is needed.

To overcome this problem and to reduce the speed variation with load, the engine is fitted with a governor. This is a device which:

- (i) can be set by the operator to give different engine speeds
- (ii) automatically increases the fuel to the engine as the load on it increases, to keep its speed approximately constant



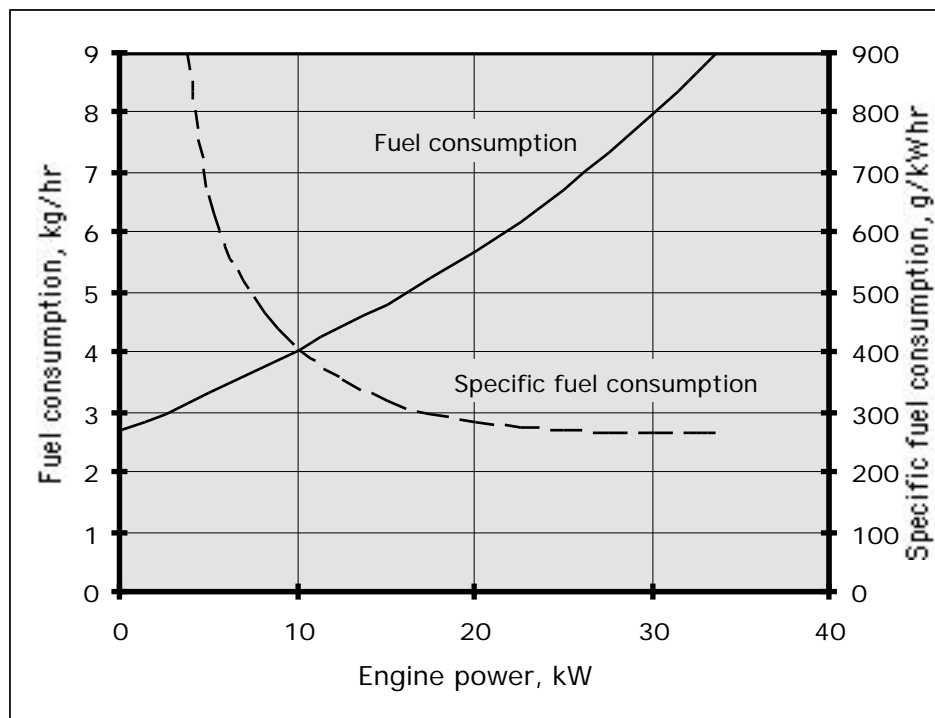


Figure 3.3 Variation of fuel consumption and specific fuel consumption with engine power for the Farmland tractor engine at maximum governor setting.

For any given governor setting, there are two ranges in which the engine can operate (Figure 3.2).

- (i) In the "governed range(s)", where the engine runs under control of the governor. As the torque load varies, so fuel is varied to keep the speed approximately constant as shown by the near vertical line. Only the maximum governor setting is shown in Figure 3.2; lines for other governor settings are shown in Figure 3.4.
- (ii) In the 'full-fuel range', where the governor is not controlling the fuel supply. The fuel system supplies a fixed maximum quantity of fuel per stroke (as set by the manufacturer); the speed varies widely (from 2250 to 1000 rpm) as shown by the dotted line in Figure 3.2.

The governed range is where the tractor is normally operated; the load and, as shown later, particularly the gear ratio are chosen to cause the engine to operate in this range. Thus the speed range is determined by the setting of the governor by the operator; within that range, the speed is automatically set by the governor.

Maximum torque for a diesel engine is reached at quite a low speed. The increase in torque as the engine slows down in the full fuel range (sometimes called "torque back-up") is a reserve of effort; it indicates the ability of the engine to increase its torque output, above that at maximum power, prior to stalling (stopping). This feature appears in the drawbar characteristics of the tractor as discussed in Section 3.3.1 and following.

(b) Power - speed

While the torque represents a fundamental performance parameter for the engine, the operator is usually more interested in the rate at which that torque effort will do work, ie, the power of the engine.

From Equation 2.3

$$\text{Engine power}^{(1)}, Q_e = 2\pi \cdot \text{Engine torque } T_e \cdot \text{Engine speed } N_e$$

For each point on and under the torque - speed curve, there is a corresponding point on and under the power - speed curve (Fig. 3.2). As the load on the engine is increased, the condition where the governor first provides the maximum fuel rate, gives maximum power for that governor setting. At higher torques and lower speeds in the full fuel range the power is less.

The output from the PTO also reflects that of the engine. However Figure 3.2 shows only one value of the power output from the PTO when it is operating at the (arbitrarily) defined 'standard PTO speed' of 540 rpm. At this speed the engine in the Farmland tractor is rotating at 1810 rpm. From this it will be seen that greater (or lesser) maximum power can be taken from the PTO but they will be at a speed greater (or less) than 540 rpm.

(c) Summary

As we increase the torque load on the engine:

- (i) in the governed ranges, the torque and power increase and the speed decreases slightly until the power reaches a maximum
- (ii) in the full-fuel range, any further increase in the torque load causes:
  - \* a small increase in the torque
  - \* a large decrease in the speed
  - \* a resultant decrease in the power
- (iii) at maximum torque the engine will stall.

Varying the governor setting:

- (i) varies the governed range of speed in which the engine runs
- (ii) varies the maximum power developed by the engine
- (iii) does not vary the maximum torque developed by the engine

The governed ranges are of most interest to the operator because it is in these that the engine operates most of the time.

<sup>(1)</sup> Engine power is often termed 'brake' power (measured by a 'brake' or dynamometer) or 'shaft' power (available at the output 'shaft').

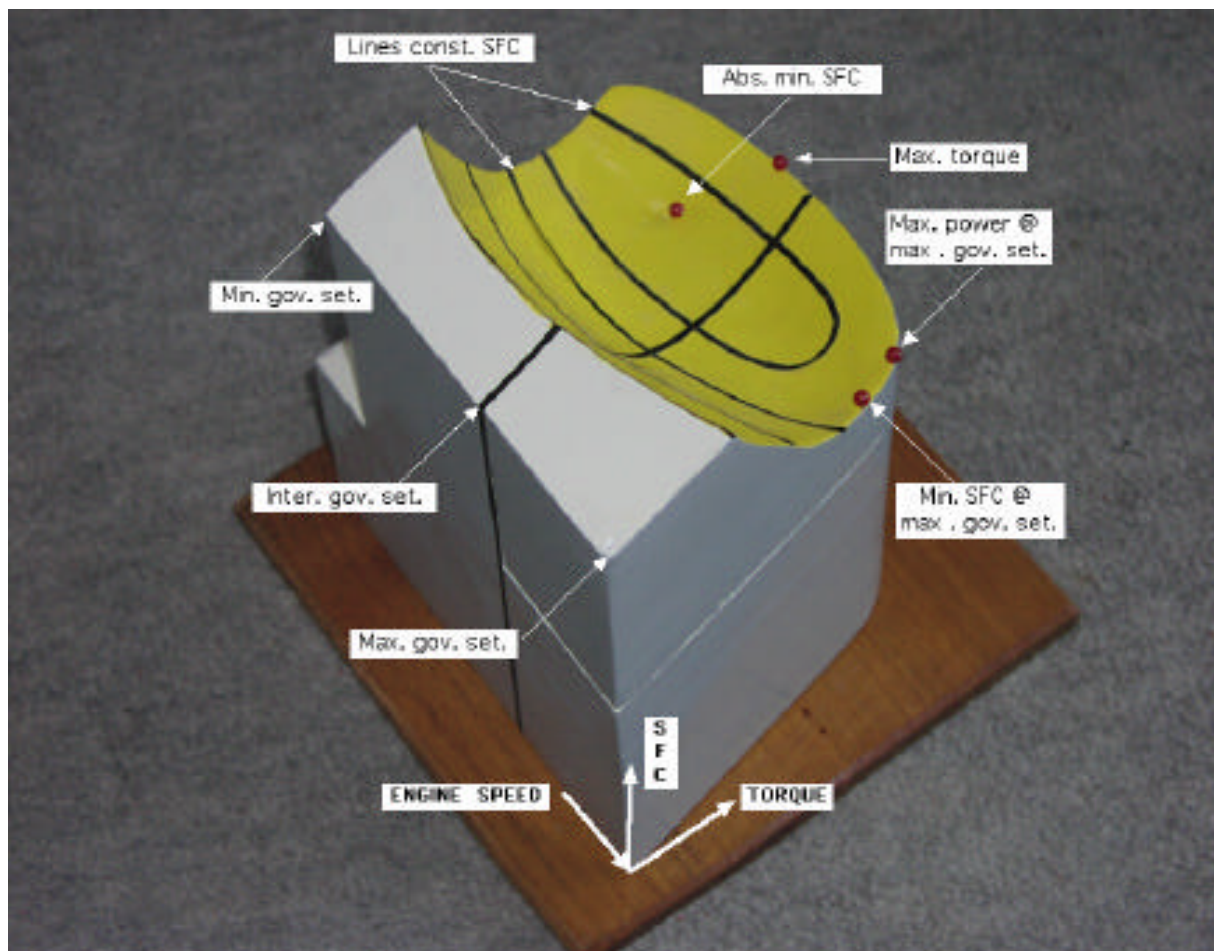
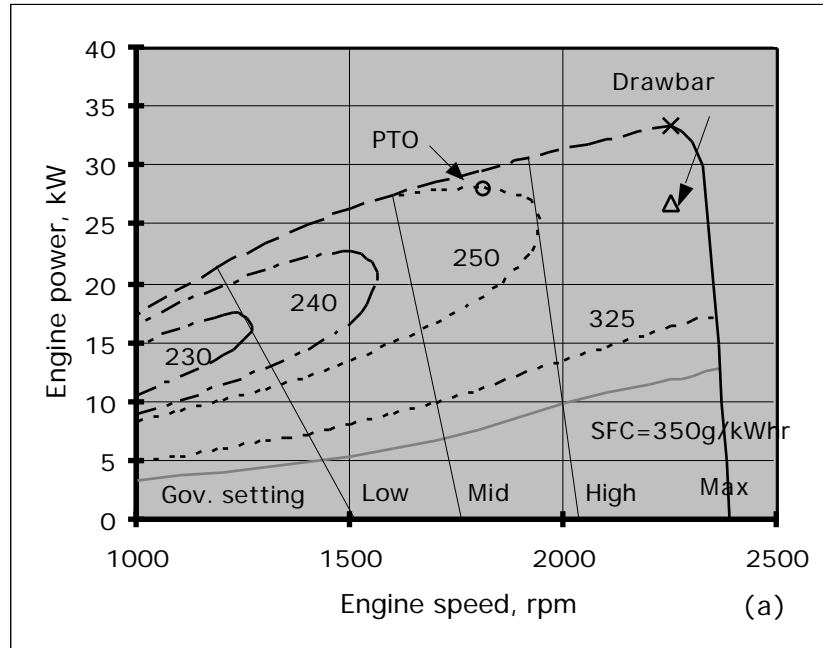


Figure 3.4: (a) Specific fuel consumption plotted on an engine power / engine speed base for the Farmland tractor engine at various governor settings.

(b) Model showing specific fuel consumption plotted on an engine torque / engine speed base for the International 434 tractor engine at various governor settings.

### 3.2.3 Input

#### (a) Fuel consumption

The other factor of interest in engine performance is the input as represented by the fuel consumption (strictly fuel consumption rate) and how this varies with the output as represented by the power in the governed range.

$$\text{Fuel consumption (FC)} = \frac{\text{Fuel used } F}{\text{Time taken } t} \quad (3.1)$$

It is quoted in kg/hr or L/hr and is usually plotted against power.

As seen in Figure 3.3 the fuel consumption (above that required to keep the engine running at zero power) is approximately proportional to power. The graph shown applies to maximum governor setting; lower governor settings would give similar, but slightly lower fuel consumption - power graphs.

#### (b) Specific fuel consumption rate

The fuel consumption is a suitable parameter for representing the input performance of one engine but does not allow a comparison of engines of different size. To do that, it is convenient to calculate the fuel consumption (rate) per kW of power developed by the engine. Hence we define:

$$\text{Specific fuel consumption (SFC)} = \frac{\text{Fuel consumption FC}}{\text{Engine power } Q_e} \quad \text{g/kWhr} \quad (3.2)$$

Specific fuel consumption (sometimes termed fuel economy) is also usually plotted against engine power as also shown in Figure 3.3; low values signify good economy, ie, low rate of fuel consumption per unit power developed.

Figure 3.3 gives the specific fuel consumption at maximum governor setting; lower governor settings would give similar, but usually slightly lower, specific fuel consumption - power graphs.

At each point on and under the power-speed graph, we can calculate a specific fuel consumption; if this is plotted perpendicular to the page we obtain a surface representing the three important aspects of the engine performance on one graph, viz, speed, power and specific fuel consumption. Lines of equal specific fuel consumption are shown as contours on Figure 3.4 (a). A model of the specific fuel consumption, here plotted on a torque - speed base, is shown (for a different tractor) in Figure 3.4(b).

The specific fuel consumption is generally lowest at 80 - 90% of maximum power at any governor setting. Hence, leaving aside other considerations discussed later, it would be desirable, from an economic point of view, to load the engine so that its operating point was in this region. The absolute lowest specific fuel consumption usually occurs at an intermediate governor setting.

#### **Problem 3.1**

An engine rotates at 2100 rpm and develops a torque of 79 Nm; it uses 1.17 kg of fuel in 15 min. Calculate the power it develops, its fuel consumption and specific fuel consumption.

Answer:

$$Q = 2\pi NT = \frac{2 \times 2100 \times 79}{60} = 17.4 \text{ kW}$$

$$FC = \frac{F}{t} = \frac{1.17}{0.25} = 4.68 \text{ kg/hr}$$

$$SFC = \frac{FC}{Q} = \frac{4.68}{17.4} = 269 \text{ g/kWhr}$$

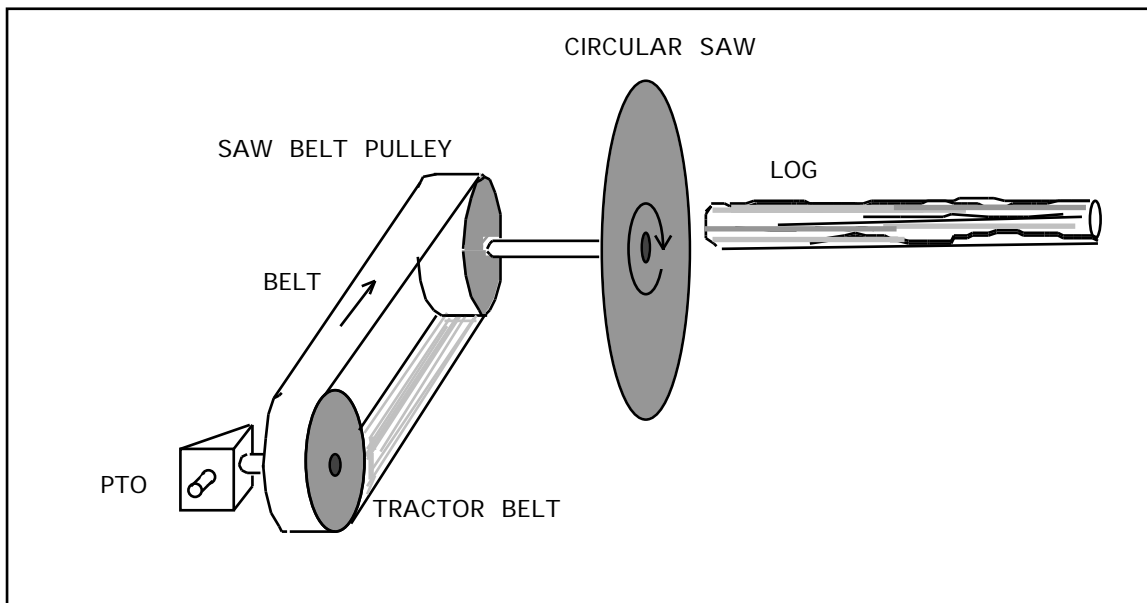


Figure 3.5 Schematic of wood saw driven from PTO of Farmland tractor; refer Problem 3.4

**Problem 3.2**

Using data for the Farmland tractor engine from Figures 3.2, 3.3 and 3.4(a):

- (i) What is the maximum power available at 2000 rpm? Answer: 31.5 kW
- (ii) What is the power available at maximum torque? Answer: 22.5 kW
- (iii) What are the FC and SFC at 25kW and maximum governor setting? Answer: 6.8 kg/hr; 270 g/kWhr
- (iv) What are the SFC and FC at 15kW and 1500 rpm?  
Answers: SFC = 245 g/kWhr (interpolated between contours)  
FC =  $245 \times 15 = 3.7$  kg/hr
- (v) What is the best SFC for a no-load speed of 2040 rpm? Answer: 250 g/kWhr
- (vi) What is the FC and SFC when the PTO is operating at 540rpm and maximum power?  
Answers: SFC = 250 g/kWhr; FC =  $250 \times 28 = 7$  kg/hr

**Problem 3.3**

Using data for the Farmland tractor engine from Figure 3.2

- (i) What is the speed of the PTO when the engine is rotating at its maximum speed and power?  
Answer: PTO speed =  $540 \frac{2250}{1810} = 670$  rpm
- (ii) Estimate the maximum power available at the PTO for maximum engine speed.  
Answer: 31.5 kW
- (iii) At what speed should the engine rotate to give a PTO speed of 600 rpm?  
Answer: PTO speed =  $1810 \frac{600}{540} = 2010$  rpm
- (iv) Estimate the efficiency of the PTO.  
Answer: PTO efficiency =  $\frac{28}{30} \times 100 = 93\%$

**Problem 3.4**

A circular saw 1.05 m in diameter is to be driven from the PTO of the Farmland tractor as shown schematically in Figure 3.5. The linear speed of the cutting tip of the saw is to be approximately 50 m/s. The pulley on the PTO gear box and that on the saw shaft are both 230 mm diameter. The belt pulley runs at 1300rpm when the engine speed is 2250 rpm.

- (i) What engine speed should be used?

Answer:

$$\text{For the saw, } V = 50 = \pi D N, \quad N = \frac{50 \times 60}{3.14 \times 1.05} = 910 \text{ rpm}$$

$$\text{For this saw speed, engine speed, } N_e = \frac{910}{1300} \times 2250 = 1575 \text{ rpm}$$

- (ii) Using Figure 3.4 (a), estimate the maximum power available at the saw. Answer: 25 kW

- (iii) If the saw absorbs 20 kW for 30% of the time and 7.5 kW for the remainder, estimate the average fuel consumption?

Answer: At 20 kW, SFC = 240 g/kWhr; FC =  $240 \times 20 = 4.8$  kg/hr

At 7.5 kW, SFC = 325 g/kWhr; FC =  $325 \times 7.5 = 2.5$  kg/hr

$$\text{Average FC} = 0.3 \times 4.8 + 0.7 \times 2.5 = 3.2 \text{ kg/hr}$$

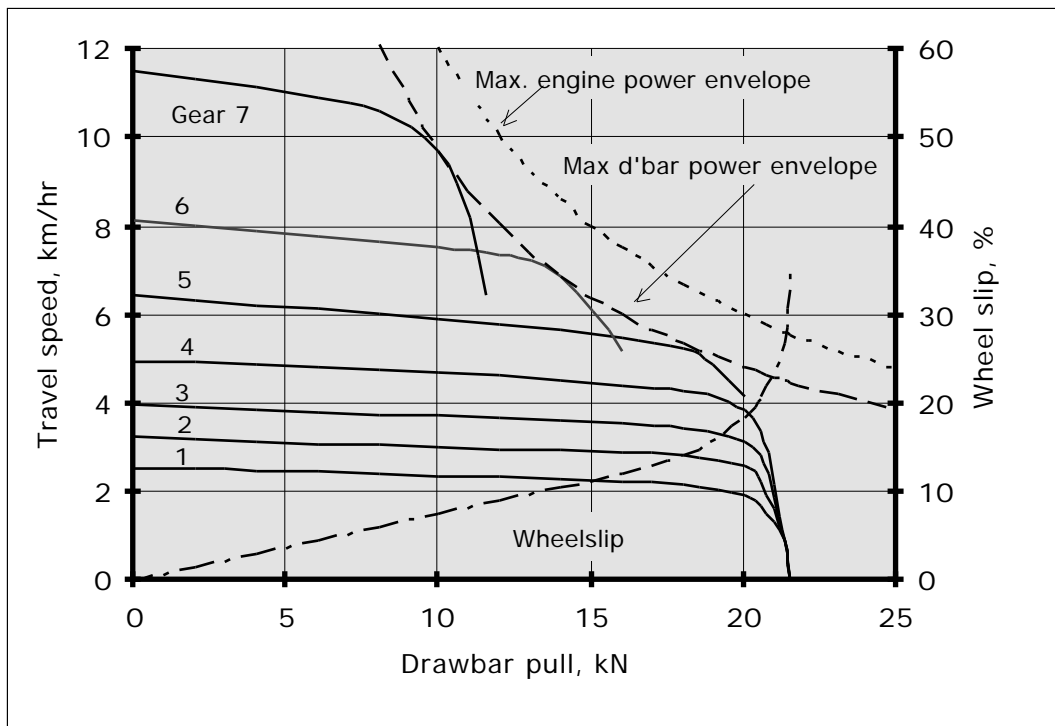


Figure 3.6 Travel speed and wheelslip versus drawbar pull for the Farmland tractor at maximum governor setting in various gears

### 3.3 TRACTOR DRAWBAR PERFORMANCE

#### 3.3.1 Output

##### (a) Travel speed - drawbar pull

The mechanism of the tractor (the transmission and wheels) converts the rotary motion of the engine to linear motion of the drawbar. As shown in Section 2.2.1 above, the tractor operates:

- (i) with an ideal travel speed:

$$V_o = D \frac{N_e}{q}$$

This neglects loss in travel speed due to slip of the driving wheels.

- (ii) with an ideal drawbar pull:

$$P = \frac{2 q T_e}{D}$$

This neglects loss in drawbar pull due to rolling resistance of the wheels.

Thus for the tractor in:

- (i) higher gears (smaller values of  $q$ , smaller speed reductions, smaller torque multiplications) will give higher travel speeds and lower maximum drawbar pulls
- (ii) lower gears (larger values of  $q$ , larger speed reductions, larger torque multiplications) will give lower travel speeds and higher maximum drawbar pulls

The actual travel speed - drawbar pull graphs for the Farmland tractor when tested on a test track at maximum governed speed are shown in Figure 3.6. Consideration of the above equations and Figure 3.2 will show that:

- (i) travel speed at zero drawbar pull is determined by gear ratio,  $q$
- (ii) travel speed decreases as drawbar pull is increased because of decreasing engine speed and increasing wheelslip

Comparison of these with the ideal graphs in Figure 2.2(c) shows that they are similar in form but the:

- (i) actual travel speeds are less than the ideal, particularly at higher drawbar pulls
- (ii) actual drawbar pulls are less than the ideal

Increasing the drawbar pull of the tractor in the three highest gears will eventually bring the engine to its maximum torque condition at which forward motion will cease; the engine will stall.

In the four lowest gears, the torque multiplication ( $q$ ) is so great that, instead of stalling the engine as in the higher gears, the engine can make the wheels slip completely and hence the drawbar pull is effectively limited by wheelslip. In these gears, the engine does not reach full power; all such gears have the same maximum pull (Figure. 3.6).

Plotting the maximum engine power envelope from Section 2.2.4 and a maximum drawbar power envelope on these axes shows how the actual performance falls short of the maximum, particularly at large drawbar pulls.

The above graphs are shown for maximum governor settings: lower settings will give lower travel speeds but approximately the same maximum drawbar pull in any gear corresponding to maximum torque, which is independent of the governor setting.

Note the small, 'triangular' shaped areas between the performance lines for the gears and the maximum drawbar power envelope. These are areas in which the engine could operate the tractor but which are unavailable because of the discrete values of the gear ratios. More gears would reduce the size of these; in the limit a continuously variable transmission (as in a hydrostatic drive) would allow the tractor to operate at all points on or under the maximum drawbar power envelope.



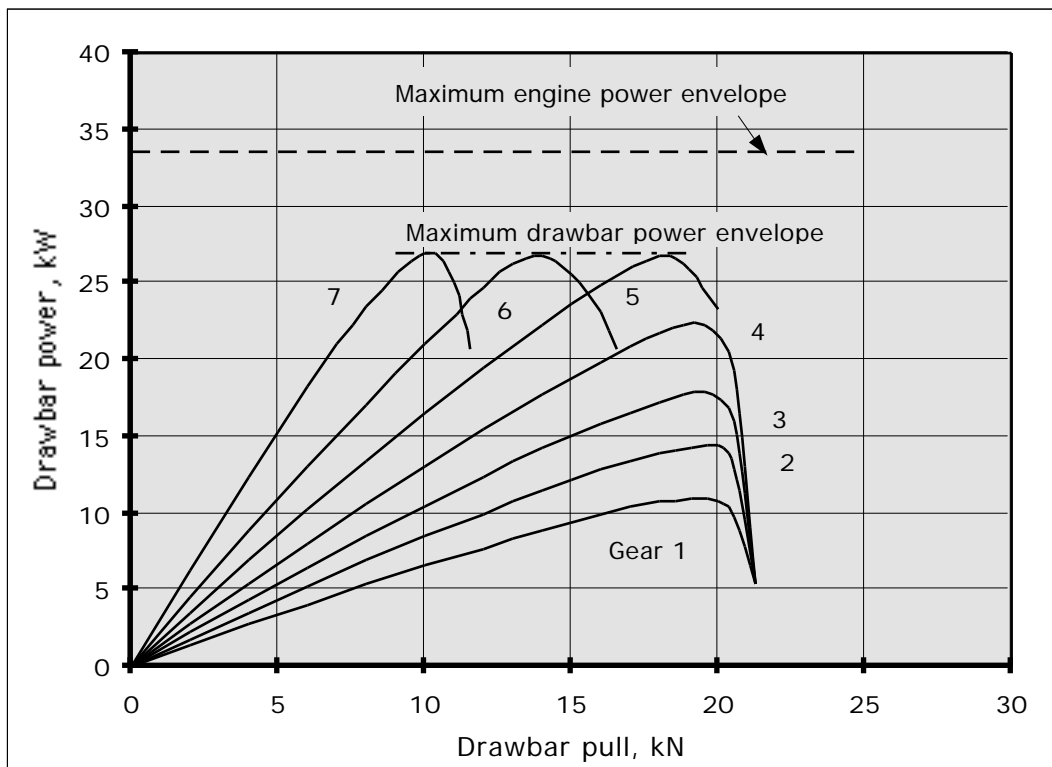


Figure 3.7 Drawbar power versus drawbar pull for the Farmland tractor at maximum governor setting in various gears.

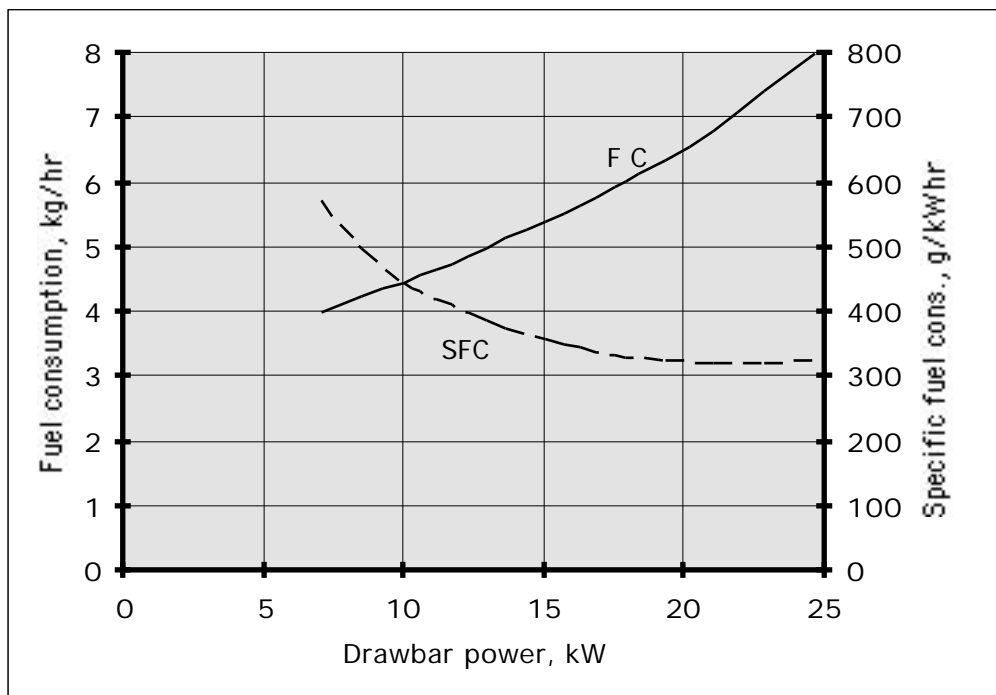


Figure 3.8: Drawbar fuel consumption and specific fuel consumption versus drawbar power for the Farmland tractor in 6th gear at maximum governor setting

(b) Drawbar power - drawbar pull

Given the drawbar pull - travel speed characteristics of the tractor shown above, the drawbar power - drawbar pull characteristic will be determined from Equation 2.4:

$$\text{Drawbar power } Q = \text{Drawbar pull } P \cdot \text{Travel speed } V$$

It is usual to plot drawbar power against drawbar pull as shown in Figure. 3.7.

Consideration of the above equation and Figure 3.6 will show that:

- (i) at zero drawbar pull, the drawbar power will be zero
- (ii) the maximum drawbar power (shown with 'x' for the three higher gears) will correspond to maximum engine power
- (iii) for the lower gears, in which wheelslip is limiting, drawbar power will not reach the value corresponding to maximum engine power

We can also identify the ideal power 'envelope' from Section 2.2.4 which the drawbar power curves approach in the higher gears. In the lower gears, where drawbar pull is limited by wheelslip, they fall far short; the difference represents mainly the power losses because of wheelslip and, to a lesser extent, rolling resistance. This matter is discussed further in Section 5.4.2.

### 3.3.2 Input

(a) Fuel consumption - drawbar power

The fuel consumption characteristics of the tractor shown in Figure 3.8 for 6th gear and for the maximum governor setting will reflect the fuel consumption characteristics of the engine. Again the fuel consumption (rate) (above that required to keep the tractor moving with no drawbar pull) is approximately proportional to the drawbar power being developed.

(b) Specific fuel consumption - drawbar power

The specific fuel consumption (rate) for the tractor is defined as:

$$\text{SFC} = \frac{\text{Fuel consumption FC (tractor)}}{\text{Drawbar power } Q}$$

The graph of specific fuel consumption versus drawbar power at maximum governor setting is also shown in Figure. 3.8.

For a given engine power the tractor SFC will be higher than for the engine alone since the drawbar power will be less than the engine power due to power loss in the transmission and wheels.

Conditions of efficient fuel use (good economy, low SFC) by the tractor will correspond to governor setting (hence engine speed), gear selected (hence travel speed) and drawbar pull (determined by the load) that will bring the engine to work in an area of low engine SFC as shown in Figure 3.4.

### 3.3.3 Other measures of tractor performance

#### (a) Wheelslip - drawbar pull

Wheelslip (usually abbreviated slip) represents a loss of forward motion by the tractor and an associated loss of power as discussed in Section 2.3 above. It arises because the force at the wheel / surface causes a loss of motion, ie, the tractor does not move forward an amount equal to the amount that the wheel rotates. (See also the more detailed discussion in Section 4.1 below).

The definition of slip given in Section 2.3.1 is equivalent to:

$$\text{Slip } i = \frac{m_o - m}{m_o}$$

where:  $m$  = distance traveled for given number of revolutions with drawbar pull

$m_o$  = distance traveled for given number of revolutions with zero drawbar pull

Because it is closely related to the wheel / surface reaction (parallel to the surface), which depends on the drawbar pull, it is usual to plot slip against this variable, as also shown in Fig. 3.6. Slip does not depend to a significant extent on speed, hence a single slip - drawbar pull graph is shown for all gears (travel speeds).

Slip is an important dependent variable in showing the 'state' of the traction process and will be used in Chapter 4 to define the drawbar pull for one optimum condition, that is, maximum drawbar power.

#### (b) Tractive efficiency

Tractive efficiency was defined in Section 2.4.1 as:

$$t = \frac{\text{Drawbar power } Q_d}{\text{Wheel power } Q_w}$$

If we assume power losses in the transmission from engine to the wheels of, say 10%, we can write:

$$t = \frac{Q_d}{0.9 \times Q_e}$$

Thus for the higher gears, for which we know both maximum engine power and maximum drawbar power (under the same conditions), we can calculate tractive efficiency as shown in Problems 3.5 and 3.9 below.

#### (c) Tractive coefficient

Tractive coefficient was defined in Section 2.4.2 as:

$$\text{Tractive coefficient} = \frac{\text{Drawbar pull } P}{\text{Weight on driving wheels } W}$$

The tractive coefficient can be used to estimate the maximum drawbar pull for the tractor with other weights on the wheels or for other tractors with similar tyres, etc; see Problem 3.10 below.

**Problem 3.5**

A tractor was tested on a firm surface and gave the following data.

Rear wheel weight	$W_r = 3900 \text{ kg}$	Engine power	$Q_e = 62.1 \text{ kW}$
Drawbar pull	$P = 26.2 \text{ kN}$	Fuel consumed	$F = 176 \text{ g}$
Distance, no-load	$m_o = 55.8 \text{ m}$	Time	$t = 25.8 \text{ s}$
Distance, load	$m = 46.2 \text{ m}$		

Determine the wheelslip, travel speed, drawbar power, tractive efficiency, fuel consumption and specific fuel consumption.

Answers:

$$\text{Wheelslip, } i = \frac{(m_o - m)}{m_o} = \frac{(55.8 - 46.2)}{55.8} = 17\%$$

$$\text{Travel speed, } V = \frac{m}{t} = \frac{46.2}{25.8} = 1.79 \text{ m/s}$$

$$\text{Drawbar power, } Q_d = PV = 26.2 \times 1.79 = 47 \text{ kW}$$

$$\text{Assuming transmission efficiency } \eta_r = 0.9$$

$$\text{Wheel power, } Q_w = 0.9 \times 62.1 = 55.9 \text{ kW}$$

$$\text{Tractive efficiency, } \eta_t = \frac{Q_d}{Q_w} = \frac{46.9}{55.9} = 84\%$$

$$\text{Fuel consumption rate, } FC = \frac{F}{t} = \frac{176}{25.8} = 6.8 \text{ g/sec} = 24.5 \text{ kg/hr}$$

$$\text{Specific fuel consumption, } SFC = \frac{FC}{Q_d} = \frac{6.8 \times 3600}{47.2} = 520 \text{ g/kWhr}$$

**Problem 3.6**

For the Farmland tractor operating in 5th gear at maximum governor setting, use data from Figures 3.6 and 3.7 to determine:

(i) the travel speed, drawbar power and the wheel slip if the drawbar pull is 10kN?

Answers: 6 km/hr, 17 kW, 7.5 %

(ii) what is the maximum drawbar pull in the governed range and the wheelslip under these conditions?

Answers: 18 kN, 15 %

**Problem 3.7**

For the Farmland tractor operating at maximum governor setting with a drawbar pull of 15 kN use data from Figure 3.6 and 3.7 to determine, for gears 1, 3 and 5, at what speeds it will travel, what drawbar powers will be developed and what will be the wheelslip?

Answers: Gear 1, 2.3 km/hr, 9 kW, 11 %; gear 3, 3.6 km/hr, 15 kW, 11 %; gear 5, 5.7 km/hr, 23 kW, 11 %.

**Problem 3.8**

For the Farmland tractor operating at maximum governor setting, and developing 20 kW at the drawbar, use data from Figure 3.6 and 3.7 to determine, for gears 4, 5 and 6, what drawbar pulls it will develop, at what speeds it will travel and what will be the wheel slips?

Answers: Gear 4, 16.5 kN, 3.5 km/hr, 12 %; gear 5, 12.5 kN, 5.9 km/hr, 9%; gear 6, 9.5 kN, 7.6 km/hr, 7 %

**Problem 3.9**

For the Farmland tractor operating in 6th gear at maximum governor setting, use data from Figure 3.6 and 3.7 to determine:

(i) what are the maximum drawbar power and the corresponding engine power?

Answers: 26.2 kW, 33.5 kW.

(ii) an estimate of the tractive efficiency:

Answers:

$$\text{Tractive efficiency } \eta_t = \frac{\text{Drawbar power } Q_d}{0.90 \times \text{Engine power } Q_e} = \frac{26.2}{0.9 \times 33.5} = 87 \%$$

**Problem 3.10**

For the Farmland tractor use data from Figure 3.6 and 3.7 to determine:

(i) What are the maximum drawbar pull and the maximum tractive coefficient if the weight on the rear wheels is 2570 kg.

Answer: 21.5 kN

$$\text{Tractive coefficient} = \frac{\text{Maximum drawbar pull } P_{\max}}{\text{Weight on driving wheels } W} = \frac{21.5 \times 1000}{2570 \times 9.8} = 0.85 \text{ at } 100\% \text{ wheelslip}$$

(ii) What weight would have to be added to the rear wheels of the tractor for it to have a maximum pull of 24 kN?

Answer: Assuming the same tractive coefficient at 100 % wheelslip:

$$\begin{aligned} \text{Weight on rear wheels } W &= \frac{\text{Maximum drawbar pull, } P_{\max}}{\text{Maximum tractive coefficient, } \eta_{t \max}} \\ &= \frac{24}{0.85} = 28.2 \text{ kN} = 2880 \text{ kg} \end{aligned}$$

$$\text{Weight to be added} = 2880 - 2570 = 310 \text{ kg}$$

Note: A large increase in the weight on the rear wheels will give a proportional increase in the drawbar pull but may overload the transmission components and / or cause the tractor to tip over rearwards.

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# **The Mechanics of Tractor - Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

## **CHAPTER 4**

### **TRACTOR PERFORMANCE ON SOFT SOIL - THEORETICAL**

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 4

### TRACTOR PERFORMANCE ON SOFT SOIL - THEORETICAL

#### 4.1 INTRODUCTION

##### 4.1.1 General

The study of tractor performance on soft soil is a typical agricultural engineering problem but it is part of a much larger subject that includes soil - implement and soil - vehicle mechanics in general and other applications associated with military and space vehicles<sup>1</sup>. Early work on military vehicles was mainly concerned with the prediction of "traffability", ie, if a simple penetrometer (a device for measuring the force to push a certain shape into the soil) could be used to predict whether a vehicle could traverse a particular area of ground.

More recent studies of tractor performance on soft soil have proceeded along two lines as mentioned in Section 1.4.2 (c) and (d) above, viz:

##### (a) Theoretical

The theoretical approach uses classical soil properties (cohesion (c) and angle of internal friction ( $\phi$ )) and some semi - empirical parameters to develop a model for the prediction of the tractive force (soil reaction) and drawbar pull. This approach, which provides the best understanding of the traction process and an appropriate introduction for students, will be followed here.

##### (b) Empirical

The empirical approach is one where the tractor performance is predicted purely on the basis of a correlation of cone penetrometer readings with corresponding performance measurements. Such an approach provides a ready and useful means of performance prediction but it is not suitable as a basis for understanding the traction process; a brief treatment is given in Chapter 5.

The usual approach to considering the prediction of tractor performance is to begin with the study of the performance of single wheels. The performance of the tractor is then understood as the combined interaction and performance of two or more such wheels.

##### 4.1.2 Definitions

The factors which are significant in the study of the performance of a single wheel may be defined as follows:

- (i) Vertical load or weight on the wheel, W is the vertical force through the axle.
- (ii) Travel (output) speed, V is the linear speed of driven wheel; there is usually some loss in motion due to wheel-slip; thus from Equation 2.1:

$$\text{Travel speed} < \text{Rotational speed} \times \text{Rolling radius}$$

- (iii) Rolling radius is defined in terms of the 'distance traveled per revolution' / 2 under some defined zero conditions; these usually include zero drawbar pull, zero braking torque and a defined surface.
- (iv) Wheel-slip, i is the proportional measure by which the actual travel speed of the wheel falls short of (or exceeds) the "theoretical" speed (Equation 2.5).
- (v) Input torque is the (rotational) input effort on driven wheel which is converted to (linear) output effort (force or drawbar pull); there is usually some loss in effort due to the rolling resistance hence from Equation 2.2:

$$\text{Drawbar pull} < \frac{\text{Input torque}}{\text{Rolling radius}}$$

<sup>1</sup> Other terms used to describe the general field include 'off-road locomotion' and 'terra-mechanics' (earth mechanics).



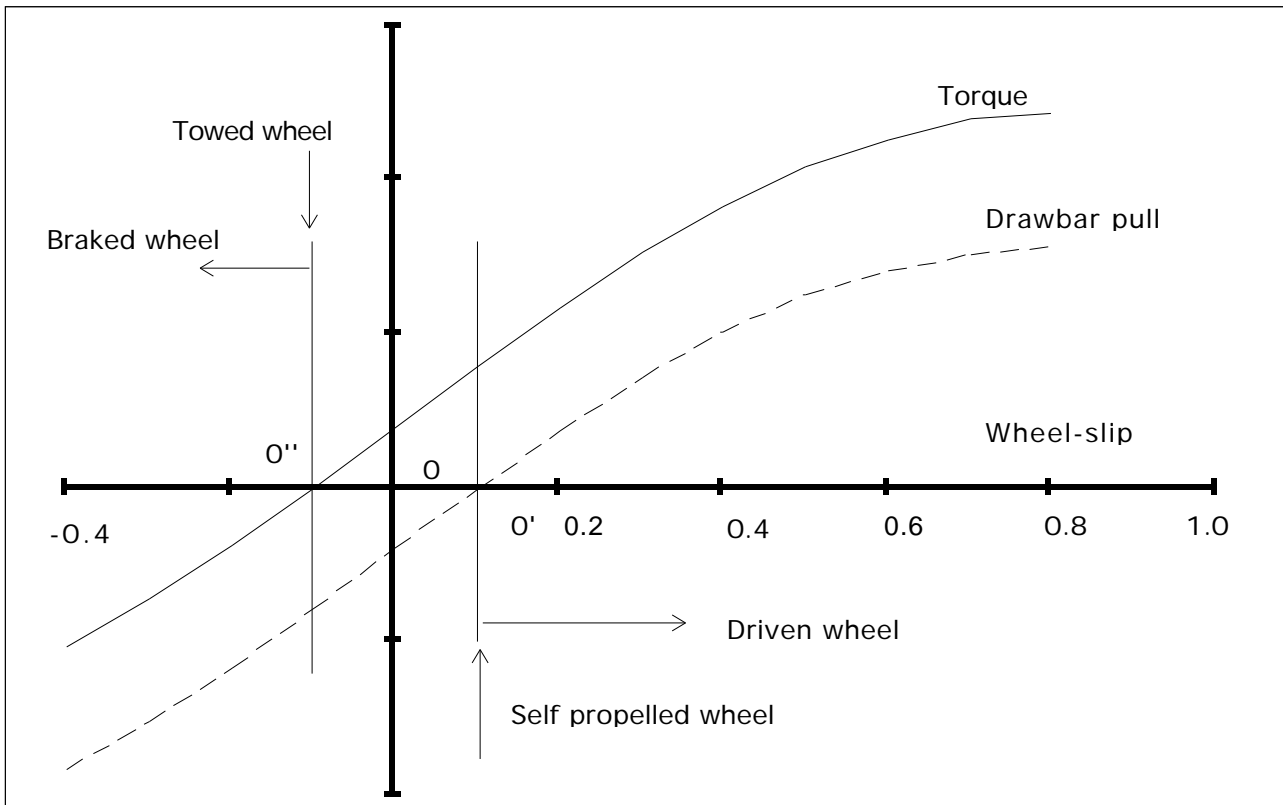


Figure 4.1: Operational states for a wheel; reproduced from Wismer and Luth (1974) with permission of American Society of Agricultural Engineers

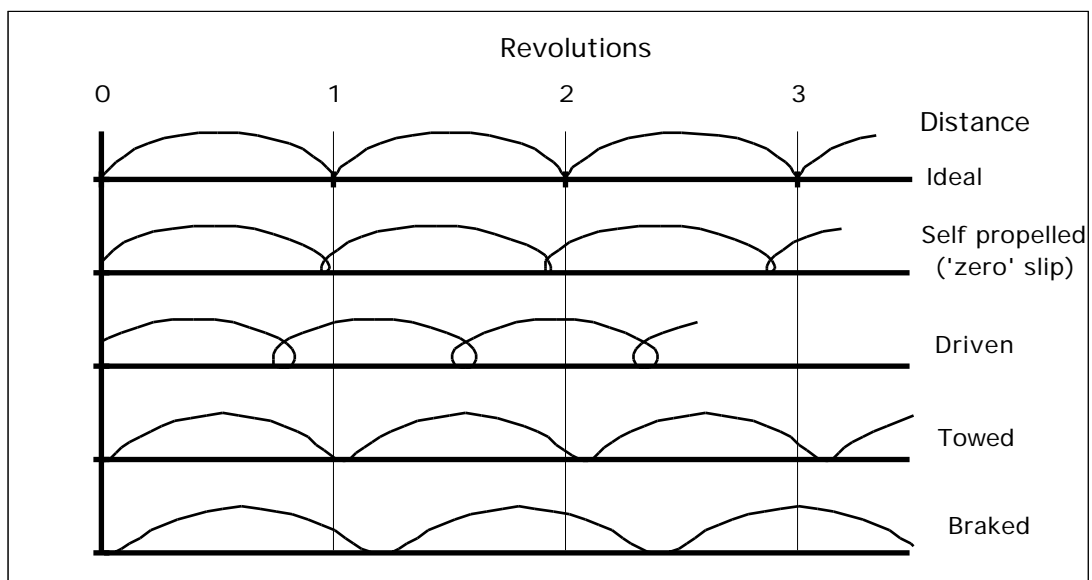


Figure 4.2: Motion of a point on a rolling wheel illustrating various conditions of slip; not to scale

- (vi) Rolling (motion) resistance,  $R$  is the force opposing motion of the wheel that arises from the non-recoverable energy expended in deforming the surface and wheel. It is convenient to consider this force as acting in the horizontal direction.
- (vii) Tractive force,  $H$  is the horizontal reaction on a driven wheel by the soil in the contact area; it is equal and opposite to the horizontal force generated by the wheel on the soil.
- (viii) Drawbar pull,  $P$  is the horizontal force at the axle generated by a driven wheel; from Equation 2.7 it may be assumed that

$$\text{Drawbar pull} = \text{Tractive force} - \text{Rolling resistance}$$

- (ix) Towing force is the force to move a freely rolling wheel over the surface and is equal and opposite to the rolling resistance.

The traditional four-wheel tractor is a combination of driven (or occasionally braked) wheels at the rear and free-rolling, towed (pushed) wheels at the front.

#### 4.1.3 Operational states of a wheel

The operation of a wheel can be classified into one of the following states; each occurs within the tractor or other machines under some conditions and each has a particular unknown parameter associated with it.

##### (a) Towed

Here the wheel, such as the front wheel of the tractor or the wheel of an agricultural implement, is towed with zero opposing external torque; the unknown parameter is the rolling resistance.

##### (b) Self-propelled

Here the wheel is driven with an external input torque to overcome its own rolling resistance and to propel it across the surface without developing a drawbar pull. This approximates to the drive wheel of a tractor with no drawbar pull (if we neglect the rolling resistance of the front wheels); the unknown parameter is the rolling resistance.

##### (c) Driven

Here the wheel is driven with an external input torque and is required to develop a drawbar pull as in the drive wheel of a tractor; the unknown parameter is the wheelslip. The extreme case is where the wheel slips, but does not move forward.

##### (d) Braked

Here the wheel is towed against an opposing, external torque as when being braked or when it is used to generate a torque to operate a 'ground-driven' machine such as a seed drill; the unknown parameter is the wheelslip. The extreme case is where the wheel does not rotate, but just skids across the surface.

Figure 4.1 (Wisner and Luth, 1974) shows these operational states of a wheel in which input and output torque and input and output force (towing force or drawbar pull) are shown plotted against wheel-slip. From this it will be seen that:

- (i) the self-propelled wheel is a special case of the driven wheel, with zero drawbar pull.
- (ii) the towed wheel is a special case of the braked wheel, with zero braking torque.

The origins for the graphs shown are based on the assumption that, with respect to the kinematic ideal (origin at  $O$ ), a self-propelled wheel is subject to some positive slip (origin at  $O'$ ) and a towed wheel is subject to some negative wheelslip (origin at  $O''$ ).

Figure 4.2 uses the trajectory of a point on a wheel rolling on a horizontal surface (the cycloid) to illustrate the effect of wheelslip by showing the distances traveled by the wheel for the various states discussed above and represented in Figure 4.1. The wheelslip is shown by the loop (motion of the wheel relative to the surface) in the trajectory for the self propelled and driven wheels.

#### 4.1.4 Wheel - slip definition

The generation of a drawbar pull by a wheel driven on a surface results in some relative motion at the wheel - surface interface. This reduces the forward motion of the wheel to less than the ideal value and is referred to generally as 'wheelslip' or 'slip'. In terms of measurement, prediction and presentation of tractor performance, slip is the single most important, dependent parameter.

Slip is defined as the proportional measure by which the actual travel speed (or distance) of a wheel falls short or exceeds the 'ideal' or 'zero' slip speed (or distance). The magnitude of slip is thus dependent on how the 'zero' slip is defined and measured.

The correct zero condition would be under conditions where the travel speed = linear speed of the surface of the wheel = rotational speed x rolling radius. However, since the rolling radius is impossible, or at least difficult, to measure, a more convenient zero condition and method of measuring it is used. This alternative 'zero' condition is defined as that occurring when the wheel is driven (usually over the (test) surface) with zero drawbar pull (no load), ie, in the self-propelled condition shown as the origin at point O' in Figure 4.1.

Thus as in Section 2.3.1 and Equation 2.5,

$$\text{Wheel-slip, } i = \frac{V_o - V}{V_o} 100 \% \quad (4.1)$$

where  $V_o$  = travel speed when the wheel is driven, with zero drawbar pull, on the surface  
 $V$  = travel speed when the wheel is generating a drawbar pull, on the surface

The driven condition is used, in preference to the towed (point O" in Figure 4.1), because it is usually more convenient to drive the wheel over the surface with zero drawbar pull than to tow it.

An alternative 'zero' condition for slip is where, for the zero pull test, the wheel is driven on a hard surface (such as a road), rather than on the test surface. Under these conditions the slip at zero drawbar pull on the test surface will not be zero. In describing an experiment it is necessary to state which 'zero' slip condition was used.

In measuring the performance of a tractor it is not possible to drive a wheel alone over the test surface, hence the zero slip condition is usually taken when the tractor is driven with zero drawbar pull over the test surface. The drive wheels will suffer some extra small slip in having to overcome the rolling resistance of the front wheels. Thus the 'zero' point will be even further to the right than O' in Figure 4.1.

#### 4.1.5 Wheelslip measurement

The use of velocity for measuring slip for a tractor as described above is not particularly convenient because variations in engine speed would influence the result, hence other methods have been devised. In the following it is assumed that the zero drawbar pull distance is measured on the test surface.

##### (a) Measurement of distance traveled

In terms of distances (for a given number of wheel revolutions):

$$\text{Wheelslip, } i = \frac{m_o - m}{m_o} 100 \% \quad (4.2)$$

where:  $m_o$  = distance traveled when the tractor is driven with zero drawbar pull on the surface  
 $m$  = distance traveled when the tractor is generating a drawbar pull on the surface

This is a convenient method when only a distance measuring tape is available and when the counting of whole numbers of wheel revolutions can be done visually; the tractor is tested over the same number of revolutions for both tests.

(b) Counting of wheel revolutions

In terms of numbers of wheel revolutions (for a given distance traveled):

$$\text{Wheel-slip, } i = \frac{N - N_o}{N} 100 \% \quad (4.3)$$

where:  $N_o$  = number of wheel revolutions when the tractor is driven with zero drawbar pull on the surface  
 $N$  = number of wheel revolutions when the tractor is generating a drawbar pull on the surface

This is a convenient method if equipment to measure fractions of a wheel revolution is available; the tractor is tested over the same distance in both tests.

(c) Use of a free rolling wheel

On some occasions it is desirable to be able to measure slip while moving but to do so it is necessary to avoid the requirement that the zero-pull test and subsequent with-pull tests be conducted over the same number of wheel revolutions (method (a) above) or for the same distance (method (b) above).

The use of a free-rolling wheel (such as an attached 'fifth wheel' or a tractor front wheel) as a 'non-slip' reference overcomes this problem in principle. The method involves the use of revolutions of the free wheel ( $n_o$  and  $n$ ) to infer the rear wheel revolutions under the zero-pull ( $N_o$ ) test, corresponding to the unknown distance used for the pull test (for which  $N$  revolutions were recorded). Thus from Equation 4.3,

$$\text{Wheel-slip, } i = \frac{N - \frac{n}{n_o} N_o}{N} \quad (4.4)$$

From Equation 4.4, it can be seen that the rear wheel revolutions ( $N_o$ ) for the zero pull tests are scaled by the ratio of the free wheel revolutions for the zero-pull and with-pull tests,  $n_o$  and  $n$ , to give the zero pull, rear wheel revolutions corresponding to the pull test distance.

In order to use this method it is necessary to have a wheel counter (to measure fractions of a revolution) on both the driving and the free wheel(s). It should also be noted that the free wheel revolutions are affected by speed and surface condition and so the free wheel may need to be calibrated if accurate results are to be obtained, particularly at small slips (Parkhill and Macmillan, 1984).

Modern techniques for continuously measuring slip using radar or ultrasonic sound for speed measurement are now available.

## 4.2 TRACTIVE PERFORMANCE

As discussed in Section 1.4 above, four different approaches have been taken to the study of tractor performance; three have been applied to tractive performance.

### 4.2.1 Practical / experimental measurement

The early study of the performance of tractors was limited to the experimental measurement of travel speed and wheel slip at various drawbar pulls on soils (for example, Southwell, 1964) and on test tracks (Baillie and Vasey, (1969) . The results, as discussed in Chapter 3, were intended to provide an understanding of the principles involved and a basis for comparing the relative performance that farmers might expect from the various tractors in the field.

Rolling resistance of wheels was measured by equating it to the towing force required to move different types of (mainly transport) wheels across visually described surfaces, eg. road (hard), stubble (firm), cultivated soil (soft) etc. The results were quoted on the basis of a coefficient of rolling resistance. The early work of McKibben and Davidson, (1940) was of this type.

### 4.2.2 Theoretical prediction

The theoretical prediction of tractive performance has involved the separation of the problem into two parts, viz, the prediction of:

- (i) tractive force,  $H$
- (ii) rolling resistance,  $R$

Using this approach it is assumed (Equation 2.7) that the drawbar pull ( $P$ ) is what remains of the tractive force after the rolling resistance has been overcome, ie:

$$P(i) = H(i) - R \quad (4.5)$$

where:  $P(i)$  implies that  $P$  will be determined as a function of slip

$H(i)$  implies that  $H$  will be predicted as a function of slip

While  $R$  is also a function of slip, this function is not known and hence the value for  $R$  is that measured under the towed condition or predicted using the theory in Section 4.3, both of which assume zero slip. Clearly this is only approximate because the rolling resistance under finite slips will be greater than the value measured or predicted with zero slip.

The generation of a tractive force by the tractor requires an equal and opposite horizontal reaction by the soil against the driving wheels in the contact area. This reaction force, which in effect determines the tractor performance, is predicted on the basis of the soil strength parameters ( $c$  and  $\phi$ ) and the soil deformation corresponding to various wheel slip values.

The support of the tractor requires a vertical reaction on the wheels which causes vertical deformation of the soil in the contact area. Equating the energy to deform the soil (ie. to make the rut) to the work done by the rolling resistance force provides a basis for calculation of the latter. The process is modelled by the pressure - sinkage relationship for a plate pressed into the soil; slip is considered to be zero (See Section 4.3).

### 4.2.3 Empirical prediction

Here experimental data on the drawbar pull and rolling resistance of various wheels together with a single soil parameter (the cone index obtained by measuring the force to push a cone penetrometer into the soil) are used to predict drawbar pull and rolling resistance on a purely empirical basis (Wisner and Luth, 1974) as discussed in Chapter 5.

As mentioned in Section 1.4 above the theoretical / predictive approach provides the best basis for understanding tractive performance and will be emphasised here; the other approaches may be more readily used for the immediate determination of wheel performance.

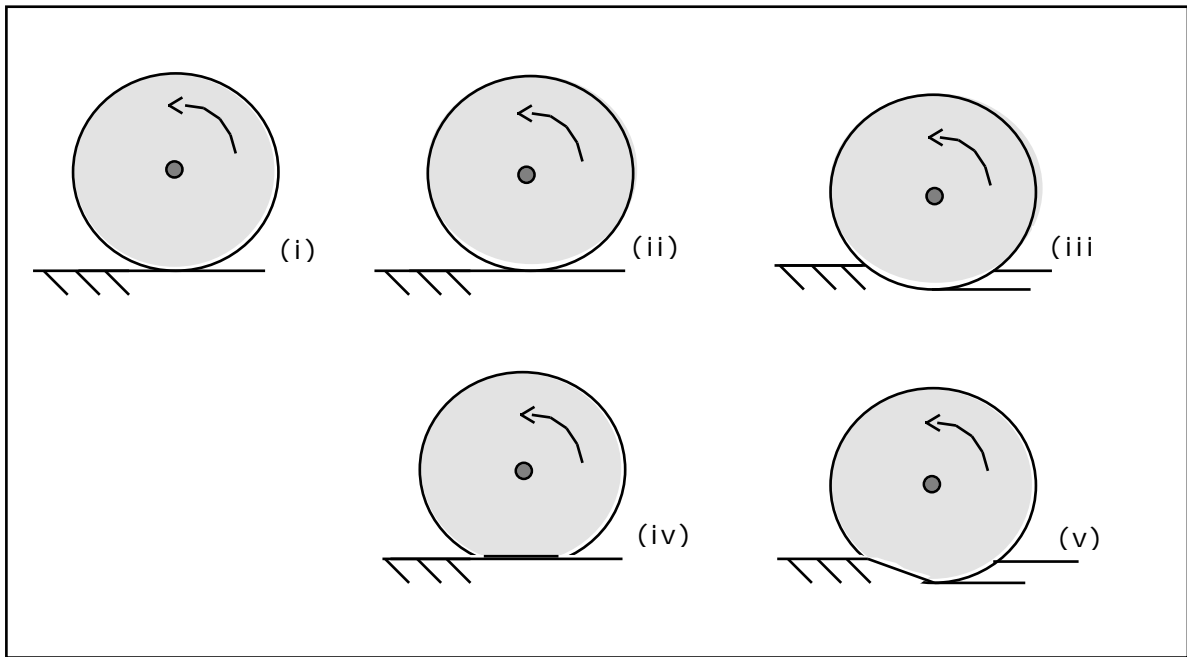


Figure 4.3: Various conditions for a wheel rolling on a surface

### 4.3 ROLLING RESISTANCE

#### 4.3.1 Wheel conditions

The rolling resistance of a wheel is, in general terms, the force opposing the motion of the wheel as it rolls on a surface. This force arises from the energy losses that occur due to

- (i) the elastic but non-ideal deformation of the wheel
- (ii) the inelastic and non-recoverable (plastic) deformation of the surface
- (iii) friction in the wheel bearings (usually assumed to be negligible)

From this it will be clear that the rolling resistance of a wheel will be a function of the strength - deformation properties of the surface and the size and deformation characteristics of the wheel. For wheels with tyres, the secondary factors include the air pressure, the structure of the tyre carcass (radial or bias ply) and the tread pattern.

For speeds used with agricultural tractors, rolling resistance is relatively independent of the speed of deformation of the soil and the tyre, hence of the travel speed.

We may consider a range of wheels as shown in Figure 4.3; here 'hard' means near rigid and 'soft' means deformable.

- (i) The ideal is a perfectly rigid wheel rolling on a perfectly rigid surface. This defines the kinematics of the rolling wheel.
- (ii) Hard wheel on a hard surface. This is approximated to by an elastic steel wheel rolling on an elastic steel track as in a railway.
- (iii) Hard wheel on soft surface. Here most of the deformation and energy loss occurs in the surface which yields plastically but does not recover. Tractor front wheels and implement wheels with 'high' pressure tyres, operating on soft agricultural soil, are of this type.
- (iv) Soft wheel on hard surface. Here most of the deformation and energy loss occurs in the wheel and appears as heat. Tractor driving wheels and vehicle wheels both operating on road surfaces are of this type.
- (v) Soft wheel on soft surface. Here both the wheel and the surface deform significantly as in the tractor rear wheel operating on soft soil. Energy loss occurs mainly in deforming the soil as in (iii) above.

One major aspect of understanding and predicting tractor performance is that of determining the rolling resistance of a wheel as it is towed without slip over the surface. The problem of determining the rolling resistance of a driving wheel, when slip is present, is more complex and will not be considered here (Reece, 1965-66).

#### 4.3.2 Theoretical prediction

When a wheel rolls over a soft surface it makes a rut or compacted track. The simplest basis for the prediction of its rolling resistance is to therefore assume that the work done against the rolling resistance is the work done in compacting the soil. Bekker (1956) assumed that the wheel was equivalent to a plate continuously being pressed into the soil to a depth equal to the depth of the rut produced by the wheel.

##### (a) Work done to deform soil

For a plate, length  $\ell$ , width  $b$ , being pressed into the soil, as in Figure 4.4, Bekker suggested that the pressure,  $p$  under such a plate is given by:

$$p = \left( \frac{k_c}{b} + k \right) z^n \quad (4.6)$$

where:

- $z$  is vertical soil deformation (sinkage)
- $k_c, k$  are soil sinkage moduli
- $n$  is soil sinkage exponent
- $b$  is the width of the plate

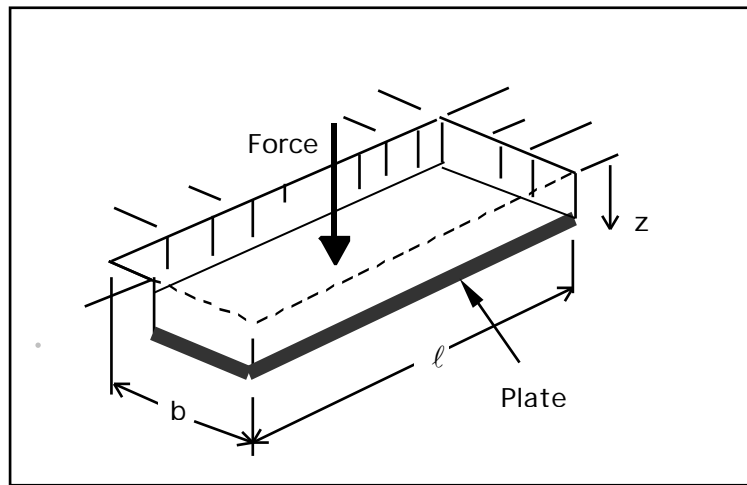


Figure 4.4: Plate being pushed into the soil to measure rolling resistance parameters (Cut away view)

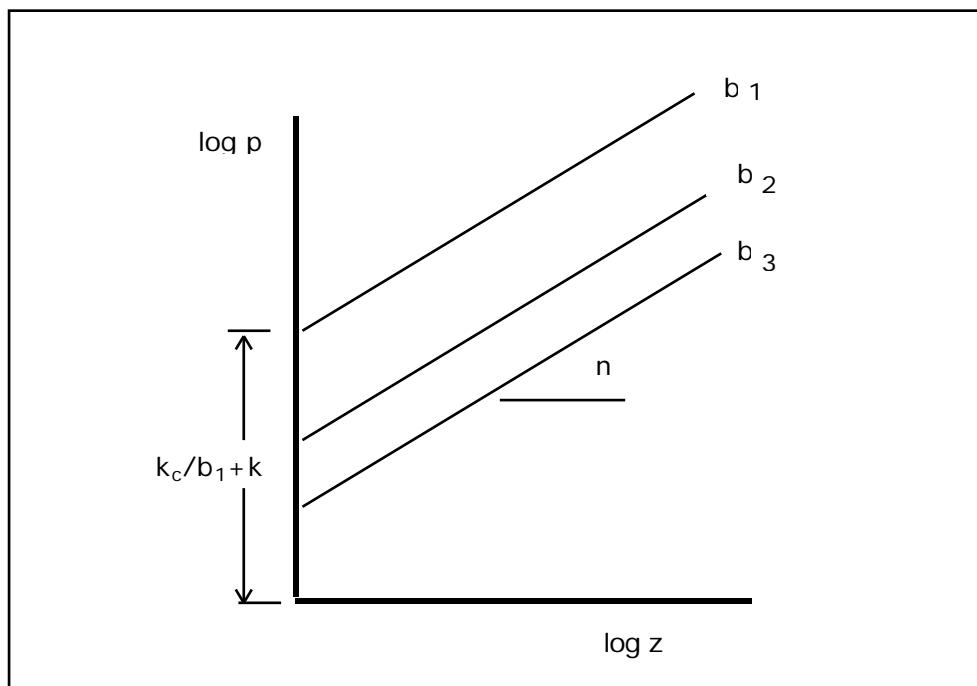


Figure 4.5: Log  $p$  plotted against  $\log z$  in analysis of plate sinkage tests.  
Reproduced from Bekker (1969) with permission of University of Michigan Press



Then the vertical work to press such a plate into the soil:

$$\begin{aligned}
 \text{Work} &= b\ell \int_0^{z_0} p \, dz \\
 &= b\ell \left( \frac{k_c}{b} + k \right) \int_0^{z_0} z \, dz \\
 &= \frac{\ell (k_c + bk)}{n+1} z_0^{n+1} \quad (4.7)
 \end{aligned}$$

But for a weight,  $W$  on the plate, at maximum sinkage  $z_0$ ,

$$\begin{aligned}
 W &= b\ell p_{\max} \\
 &= b\ell \left( \frac{k_c}{b} + k \right) z_0^n \\
 &= \ell (k_c + bk) z_0^n \\
 z_0 &= \left[ \frac{W}{\ell (k_c + bk)} \right]^{1/n}
 \end{aligned}$$

Substituting for  $z_0$  in Equation 4.7 gives

$$\text{Work} = \frac{\ell (k_c + bk)}{n+1} \left[ \frac{W}{\ell (k_c + bk)} \right]^{(n+1)/n} \quad (4.8)$$

Before considering the two types of wheel / surface that have been analysed on this basis we need to show how the soil parameters can be measured.

#### (b) Measuring soil parameters

Because the work to compact the soil is used as the basis of prediction of rolling resistance, the force to push a plate into the soil and the associated sinkage is chosen as an appropriate method of determining the soil parameters for the calculation of rolling resistance.

To obtain the parameters, a series of plates of different widths,  $b_1, b_2, b_3$  are pushed into the soil while the force and corresponding sinkage are measured. From Equation 4.6 we can write:

$$\log p = \log \left( \frac{k_c}{b} + k \right) + n \cdot \log z$$

Assuming the data follow Equation 4.6, when  $\log p$  is then plotted against  $\log z$ , we get a series of straight lines of slope 'n' and intercept on the  $\log p$  axis =  $\left( \frac{k_c}{b} + k \right)$  as shown in Figure 4.5. Further if the intercepts are then plotted against  $\frac{1}{b}$  the slope of this line is  $k_c$  and the intercept at  $\frac{1}{b}$  is  $k$ .

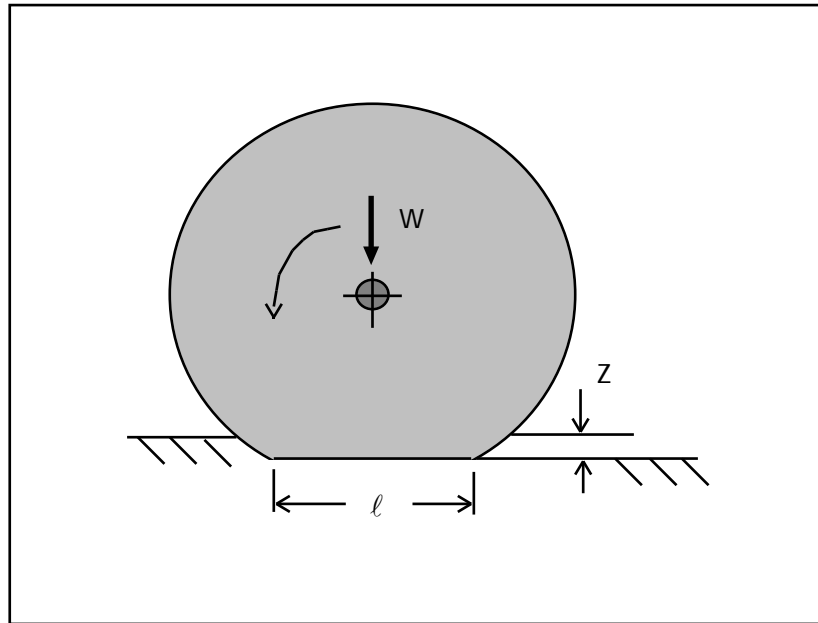


Figure 4.6: Parameters for the analysis of the rolling resistance of a soft wheel on a soft surface.  
Reproduced from Bekker (1960) with permission of the University of Michigan Press.

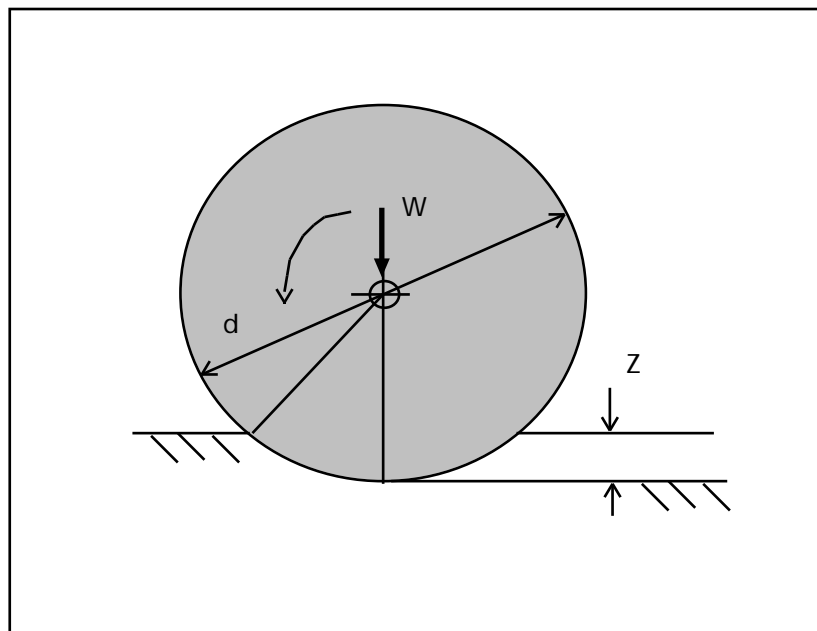


Figure 4.7: Parameters for the analysis of the rolling resistance of a hard wheel on a soft surface.  
Reproduced from Bekker (1956) with permission of University of Michigan Press.

(c) Soft wheel on soft surface

Here the wheel (or a track) is assumed to impose a uniform pressure on the soil which deforms uniformly over the contact area (as in Figure 4.6(a)) until the contact area times the pressure at the tyre surface is equal to the weight on the tyre. This pressure may be assumed to be made up of the pressure equivalent to the stiffness of the tyre carcass and the internal pressure of the air (and the water if used).

Consider the work done in towing such a wheel a distance,  $\ell$  against the rolling resistance,  $R$ . In simple terms, if this is equal to the work done on forming the rut as calculated for the plate, length  $\ell$ , width  $b$  pressed into the soil, as in (a) above:

$$R \ell = \frac{\ell (k_c + b k)}{n+1} \left[ \frac{W}{\ell (k_c + b k)} \right]^{(n+1)/n}$$

Thus the rolling resistance,

$$\begin{aligned} R &= \frac{(k_c + b k)}{n+1} \left[ \frac{W}{\ell (k_c + b k)} \right]^{(n+1)/n} \\ &= \frac{1}{(n+1)(k_c/b + k)^{1/n}} \left[ \frac{W}{\ell} \right]^{(n+1)/n} \end{aligned} \quad (4.9)$$

Writing this in terms of the ground pressure  $p = \frac{W}{b\ell}$  gives:

$$R = \frac{b}{(n+1)(\frac{k_c}{b} + k)^{1/n}} (p)^{(n+1)/n} \quad (4.10)$$

This simple analysis suggests that rolling resistance depends directly (but not necessarily proportionally) on the weight on the wheel  $W$ , and inversely (but not necessarily proportionally) on the length of the contact area,  $\ell$  but not the diameter of the wheel except in so far as it affects  $\ell$ . It also depends in a complex way on the width of the contact area,  $b$ .

For  $n = 1$ , which might be considered typical for an agricultural soil (Dwyer, 1984), this equation can be put in the form of a coefficient of rolling resistance (see Section 4.3.3):

$$\frac{R}{W} = \frac{p}{2\ell (\frac{k_c}{b} + k)} \quad (4.11)$$

This equation suggests that the coefficient of rolling resistance will be proportional to the ground pressure and inversely proportional to the length of the contact area. Hence, for example, improved traction will be achieved on sandy soils if  $p$  is small and  $\ell$  is large, ie, by the use of low pressure tyres.

d) Rigid wheel on a soft surface

Here the problem, as shown in Figure 4.6(b), is more complex because the sinkage and hence the pressure is not constant over the contact area as was assumed for the uniform sinkage case above. It can be shown (Bekker 1956) that:

$$R = \frac{3W \frac{2n+2}{2n+1}}{(3-n)\sqrt{D}} \frac{1}{(n+1)(k_c + bk)^{\frac{1}{2n+1}}} \quad (4.12)$$

Here it will be seen that the rolling resistance is dependent, in a complex way, on the weight on the wheel as well as its width and diameter compared with the length of the contact patch in the previous analysis.

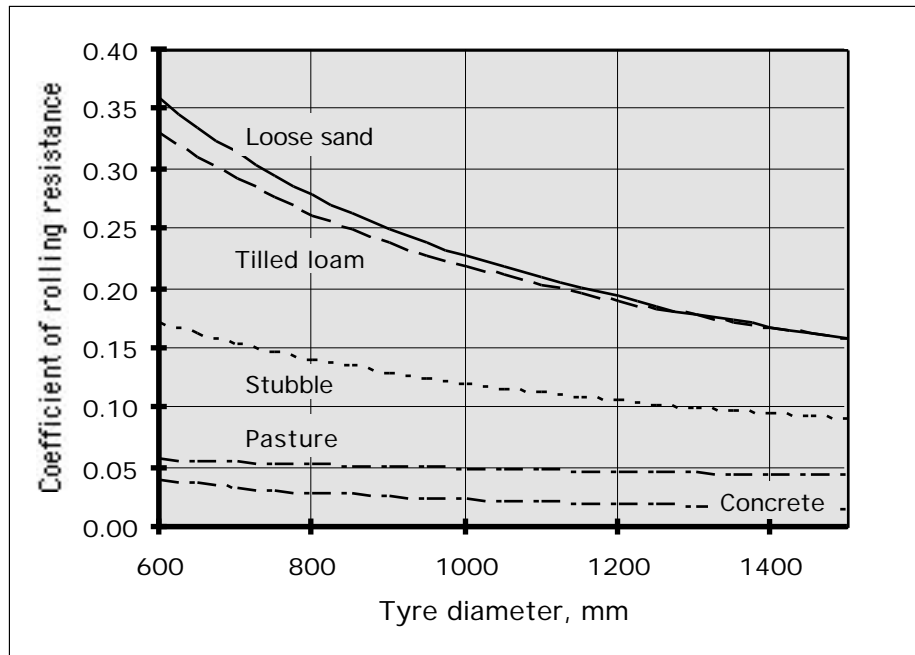


Figure 4.8: Rolling resistance of agricultural tyres of different diameter on various surfaces.  
 Reproduced from McKibben and Davidson (1940) with permission of the  
 American Society of Agricultural Engineers

### 4.3.3 Experimental measurement

Historically the experimental measurement of rolling resistance provided the data for the evaluation of traction systems. The weight on the wheel, the wheel diameter and / or width and the soil condition were seen as the most important factors and so the rolling resistance for each type of wheel was expressed in terms of the dimensionless number:

$$\text{Coefficient of rolling resistance, } = \frac{\text{Rolling resistance force}}{\text{Weight force}} \quad (4.13)$$

Use of such a coefficient requires that the wheels must be defined in terms of their diameter, width, etc, and soil conditions be verbally described.

The early work of McKibben and Davidson (1940), as shown (corrected) in Figure 4.8, used this approach. The intuitive and practical experience that we have of the significance of wheels rolling on soft surfaces is confirmed by that graph. There it will be seen that the coefficient for sand and loose soil is some 4 - 6 times that for concrete and firm soil and that doubling the diameter will halve the coefficient.

### 4.3.4 Empirical prediction

The empirical prediction of rolling resistance is considered in Chapter 5

## 4.4 TRACTIVE FORCE

### 4.4.1 Introduction

A track or wheel generates a tractive force by reacting (pushing) against the soil. Any such force involves shear stresses in, and an associated deformation between the track (together with the soil between its lugs or grousers) and the underlying soil bulk. For the track as a whole such deformation results in slip or lost motion. An analysis of the generation of tractive force therefore requires a knowledge of the stress - deformation relationship of the soil.

### 4.4.2 Shear stress - deformation characteristic for soil

The shear stress - deformation relationship for soils may take different forms depending on the normal and shear stresses under which they were compacted and their degree of cementation (bonding together of the soil particles). Bekker (1956) fitted empirical equations to two typical forms and analysed tractive force by integrating them over the length of the track. Only the simpler analysis applicable to loose and / or non-cemented soil with slowly rising shear stress - deformation characteristic (as shown in Figure 4.9) will be given here.

The soil shear stress - deformation characteristic for such a soil is assumed to have the following form:

$$S = S_{\max} (1 - e^{-j/k}) \quad (4.14)$$

where  $S_{\max}$  = shear strength of the soil and corresponds to shear stress at large deformation

$$= (c + \sigma \tan \phi)$$

$c$  = soil cohesion

$\phi$  = angle of internal friction

$\sigma$  = normal stress

$j$  = shear deformation

$k$  = shear deformation modulus

$$\text{Hence } S = (c + \sigma \tan \phi) (1 - e^{-j/k}) \quad (4.15)$$

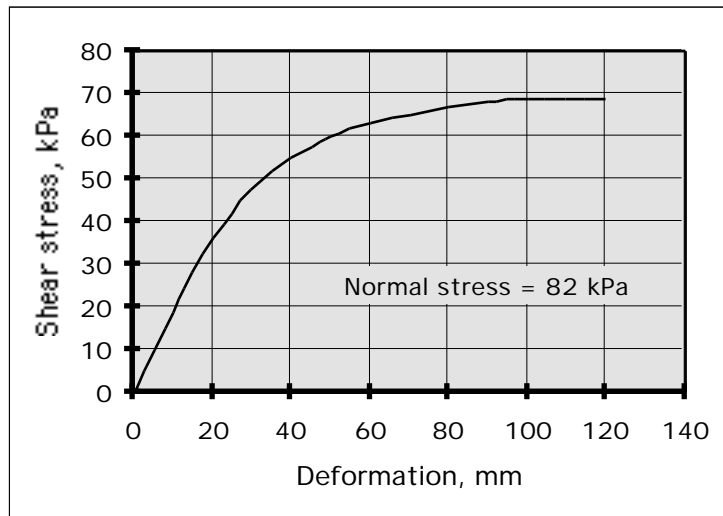


Figure 4.9: Typical shear stress / deformation curve for a loose uncemented soil.

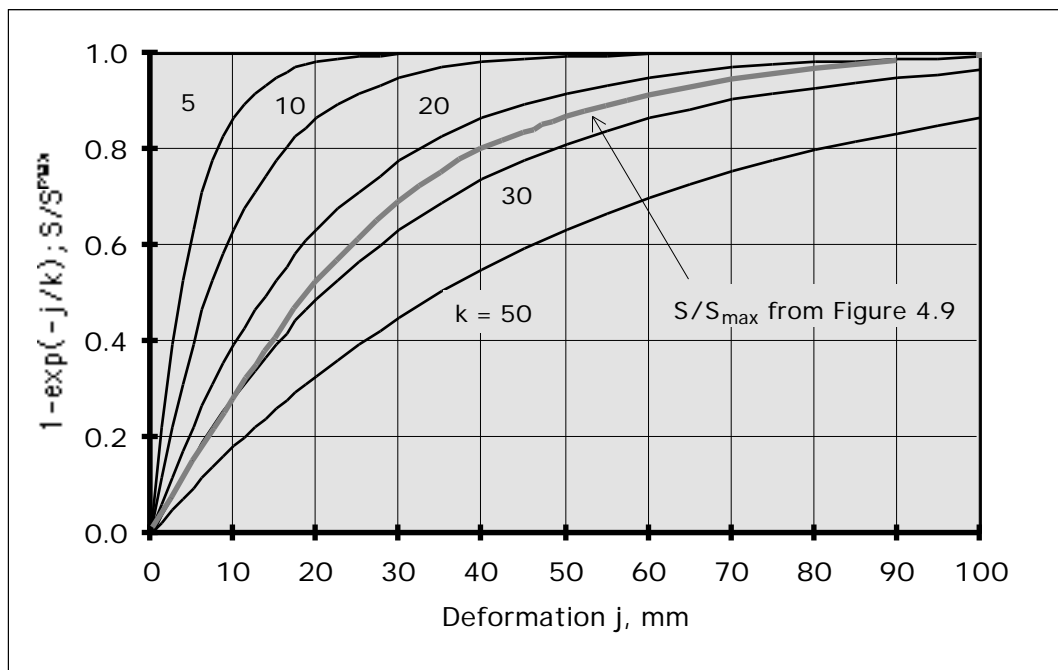


Figure 4.10: Plot of  $1 - \exp(-j/k)$  and  $S/S_{\max}$  from Figure 4.9

The shear deformation modulus indicates the 'rigidity' or deformation at which the soil reaches its shear strength in being sheared. It is a characteristic dimension and is taken as that at which the shear stress reaches 95% of its final value (Wills, 1963) as shown in Figure 4.10.

$$\text{ie. } \frac{S}{S_{\max}} = 0.95 = (1 - e^{-j/k})$$

$$e^{j/k} = 20 \quad \text{ie, } \frac{j}{k} = \ln 20$$

$$k = \frac{j}{3} \quad (\text{closely})$$

Thus  $k$  is  $1/3$  of the deformation corresponding to 95% of the maximum shear stress.

To determine  $k$ , it is necessary to measure the shear stress ( $S$ ) - deformation ( $j$ ) characteristic for the soil and then to plot the following against  $j$ :

$$(i) \frac{S}{S_{\max}} \quad \text{from experimentally measured results (Figure 4.9, Pudjiono (1998))}$$

$$(ii) 1 - e^{-j/k} \quad \text{for different assumed values of } k, \text{ as shown plotted in Figure 4.10.}$$

The modulus  $k$  may then be chosen by inspection according to the value corresponding to that graph (ii) which best fits (i). Other methods are discussed by Wills (1963).

#### 4.4.3 Analysis of locked track

Consider a rigid, inextensible track as shown in Figure 4.11 standing on a soil with strength parameters, cohesion ( $c$ ) and angle of internal friction ( $\phi$ ) and with a rising stress - deformation characteristic, as given in Figure 4.9. Assume track grousers of width  $b$ , length  $\ell$  and carrying a weight  $W$ , are engaged in the soil.

If the track is locked, the maximum tractive force that the track can generate will be the maximum force the soil can resist.

$$\begin{aligned} H_{\max} &= \text{Area } S_{\max} \\ &= b \ell (c + \tan \phi) \\ &= b \ell c + b \ell \tan \phi \\ H_{\max} &= Ac + W \tan \phi \end{aligned} \quad (4.16)$$

This neglects any contribution of the soil being sheared at the end of the grousers.

$H_{\max}$  represents the absolute maximum capacity of the track at large soil deformation corresponding (approximately) to 100% slip. According to this simple theory, it is an upper-bound value that may be approached but never exceeded.

This equation implies that  $H_{\max}$  depends on:

- (i) the area of the track which contributes to  $H_{\max}$  through the cohesive strength of the soil
- (ii) the weight on the track which contributes to  $H_{\max}$  through the frictional strength of the soil

Dividing by  $W$  gives, in a similar way to Equation 2.15,

$$\begin{aligned} \mu' &= \frac{H_{\max}}{W} = \frac{c}{\frac{W}{A}} + \tan \phi \\ &= \frac{c}{A} + \tan \phi \end{aligned} \quad (4.17)$$

where  $\mu'$  is a 'gross' tractive coefficient.

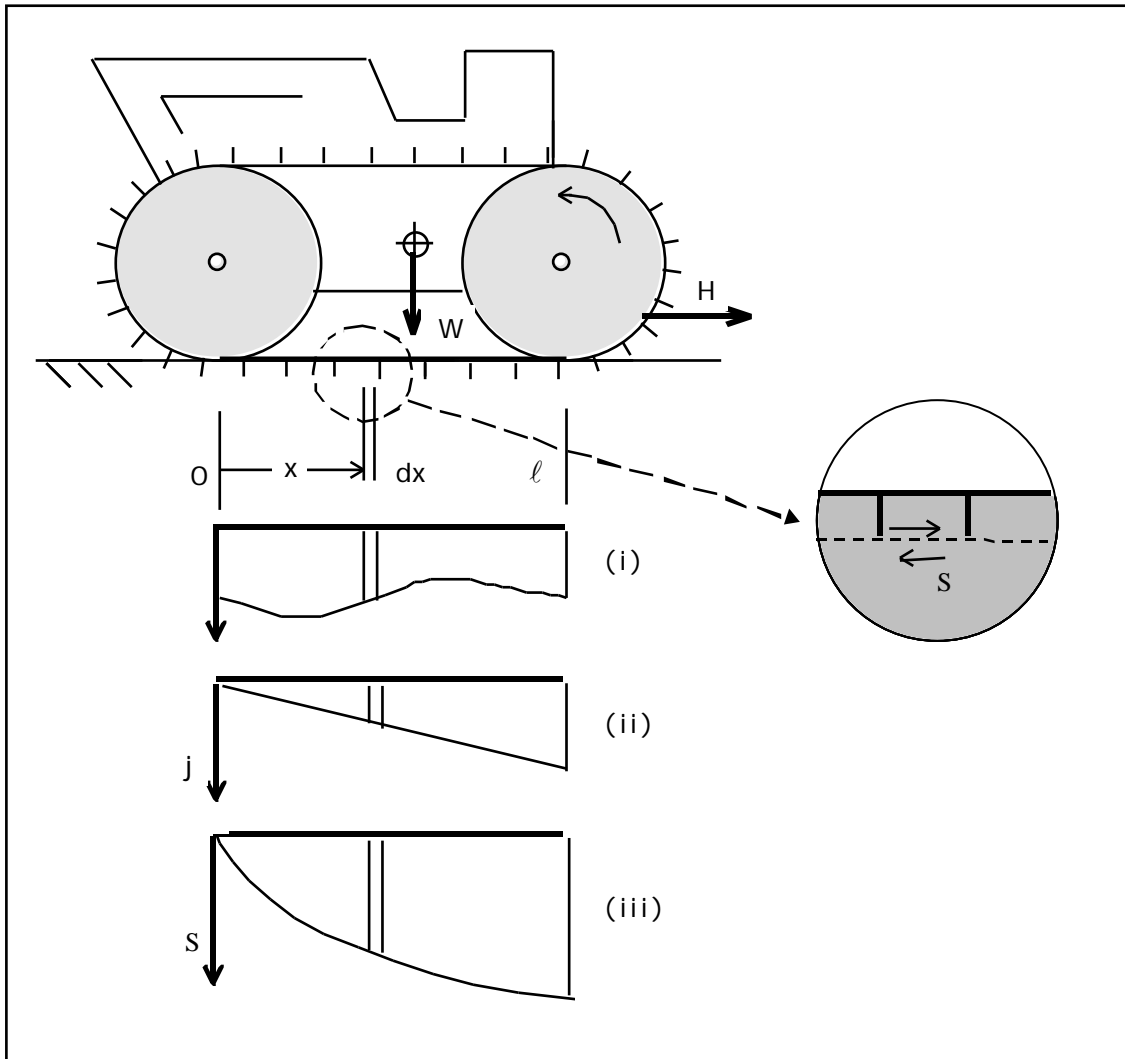


Figure 4.11 Operational parameters for a track showing the variation along the track of:  
 (i) normal stress,  $\sigma$ ; (ii) horizontal deformation,  $j$ ; shear stress,  $S$ .



**Problem 4.1**

Figure 4.12 shows a crawler tractor standing (a) on level ground and (b) on a slope.

The following data apply:

Track - soil contact length	$\ell$	= 1.2 m
Track width (total for two)	$b$	= 0.6 m
Tractor mass	$W$	= 2.4 T
Soil cohesion	$c$	= 15 kPa
Soil angle of internal friction		= $30^\circ$
Angle of slope		= $15^\circ$

Estimate the capacity,  $H$ , of the tractor as an anchor and the gross tractive coefficient,  $\mu$ ; assume that the normal stress under the track is uniform.

Solution (b):

Resolving along the slope:

$$H \cos \alpha + W \sin \alpha = Ac + (W \cos \alpha - H \sin \alpha) \tan \alpha$$

$$H \cos \alpha + H \sin \alpha \tan \alpha = Ac + W \cos \alpha \tan \alpha - W \sin \alpha$$

$$H = \frac{Ac + W(\cos \alpha \tan \alpha - \sin \alpha)}{(\cos \alpha + \sin \alpha \tan \alpha)} = \frac{1.2 \times 0.6 \times 15 + 23.5 (\cos 15^\circ \tan 30^\circ - \sin 15^\circ)}{(\cos 15^\circ + \sin 15^\circ \tan 30^\circ)}$$

$$= 16 \text{ kN}$$

$$\mu = \frac{H}{W \cos \alpha - H \sin \alpha} = \frac{16}{18.6} = 0.86$$

Answers: (a) 24.4, 1.04

Repeat for other arrangements where  $H$  is neither along the slope nor horizontal.

**Problem 4.2**

A rubber wheel carrying a load  $W$  of 5.4 kN has an effective ground contact area  $A$  of  $0.09 \text{ m}^2$  over which the pressure may be assumed to be uniform. The soil and rubber / soil strength characteristics are shown in Figure 4.13

What is the maximum pull which can be generated by the wheel if:

- the wheel has lugs which engage the soil?
- the lugs are removed?

Solution (i):

$$p = \frac{W}{A} = \frac{5.4}{0.09} = 60 \text{ kPa}$$

$$H_{\max} = \text{tractive force at the contact area}$$

$$= 36 \times 0.09$$

$$= 3.24 \text{ kN}$$

Alternatively the strength of the soil may be calculated as  $Ac + W \tan \alpha$ .

$$H_{\max} = 0.09 \times 20 + 5.4 \times 0.267 = 1.8 + 1.44 = 3.24 \text{ kN}$$

Answer (ii): 1.57 kN

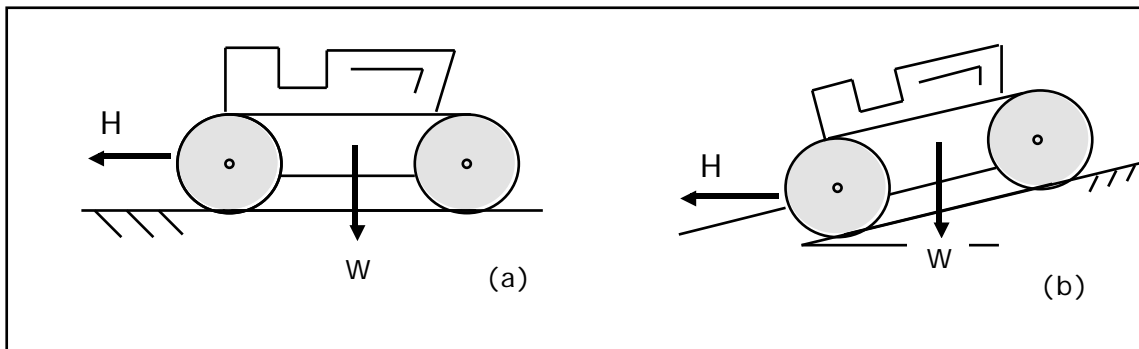


Figure 4.12: Tractor as an anchor in Problem 4.1

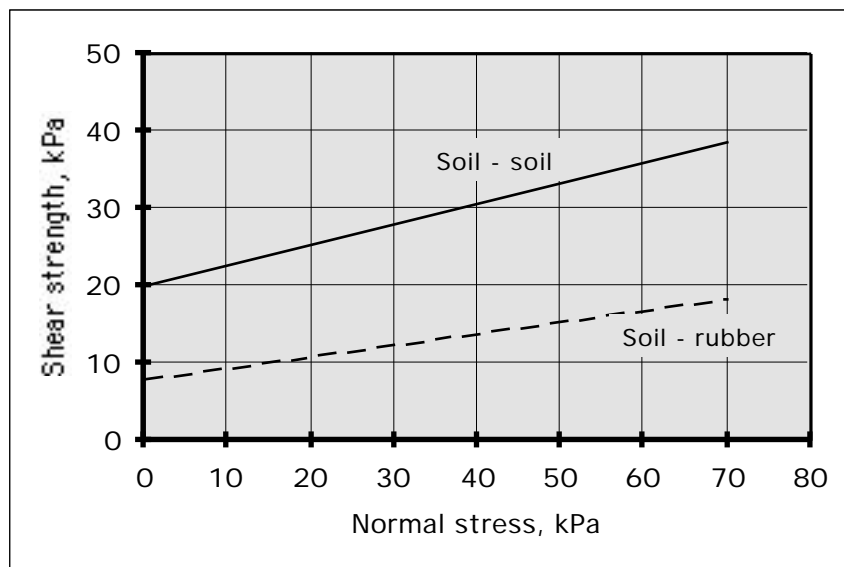


Figure 4.13: Soil and rubber characteristics for Problem 4.2

#### 4.4.4 Analysis of track with slip (Bekker, 1956)

Consider the track in Figure 4.11 being driven over a soil surface while developing a drawbar pull = tractive force  $H$ . The rotation of the track is such that a length of track equal to the track wheel centre distance ( $\ell$ ) is laid out; this is equivalent to a fraction of a revolution. Before the track moves an element of soil at its front will have zero deformation; after the track has passed over it will have a finite value,  $j_{\max}$ .

From Equation 4.2 above,

$$\text{Track slip, } i = \frac{m_0 - m}{m_0}$$

For no tractive force, the movement of the track forward will be equal to the wheel centre distance, ie.  $m_0 = \ell$

With tractive force the movement of the tractor =  $m$

$$i = \frac{\ell - m}{\ell}$$

But  $(\ell - m)$  = maximum distance moved rearwards by the soil, ie,  $j_{\max}$ .

$$i = \frac{j_{\max}}{\ell}$$

But since the track is inextensible, the deformation must grow linearly from front to rear as shown in Figure 4.11.

$$i = \frac{j}{x}$$

$$j = i x \quad (4.18)$$

Tractive force is the sum of the contributions of the shear stress (times the corresponding area) for all the elements of soil along the track :

$$\begin{aligned} H &= \int_0^{\ell} b S dx \\ &= \int_0^{\ell} b (c + (x) \tan \phi) (1 - e^{-j/k}) dx \end{aligned}$$

where  $(x)$  represents  $\phi$  as a function of  $x$ .

If it is assumed that  $\phi$  is constant, ie, independent of  $x$ ,

$$\begin{aligned} H &= \int_0^{\ell} b (c + \tan \phi) (1 - e^{-j/k}) dx \\ &= \int_0^{\ell} b (c + \tan \phi) (1 - e^{-ix/k}) dx \\ &= b (c + \tan \phi) \left[ x + \frac{k}{i} e^{-ix/k} \right]_0^{\ell} \\ &= b (c + \tan \phi) \left[ \ell + \frac{k}{i} e^{-i\ell/k} + \frac{k}{i} \right] \end{aligned}$$

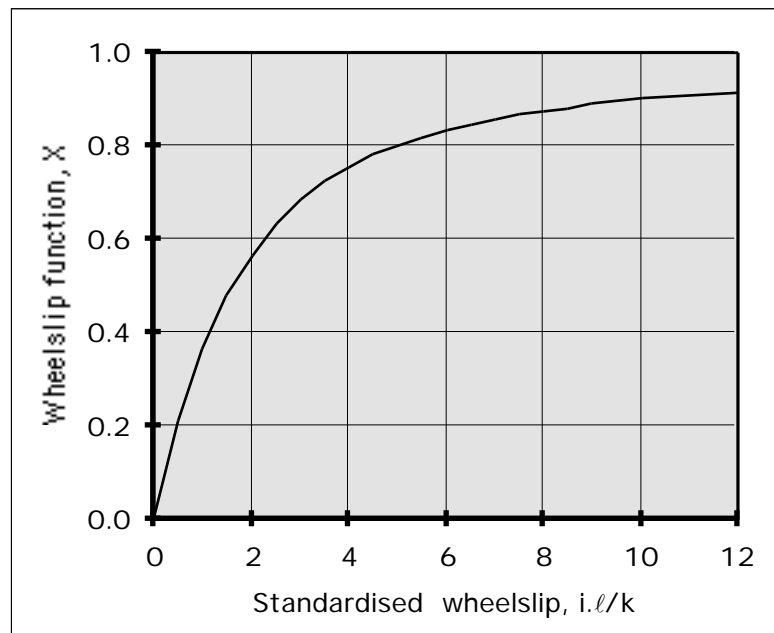


Figure 4.14: Slip function,  $X$  versus  $i.l/k$ ; reproduced from Reece (1967) with permission of the Institution of Agricultural Engineers.

$$\begin{aligned}
&= b \ell (c + \tan \phi) \left[ 1 - \frac{k}{i\ell} + \frac{k}{i\ell} e^{-i\ell/k} \right] \\
&= (Ac + W \tan \phi) \left[ 1 - \frac{k}{i\ell} + \frac{k}{i\ell} e^{-i\ell/k} \right] \\
&= H_{\max} \cdot X
\end{aligned} \tag{4.19}$$

where  $X$  = slip function for assumed constant normal stress.

$$X = \left[ 1 - \frac{k}{i\ell} + \frac{k}{i\ell} e^{-i\ell/k} \right] \tag{4.20}$$

The slip function  $X$  is shown plotted against  $\frac{i\ell}{k}$  in Figure 4.14. This is a slip - tractive force graph where  $\frac{i\ell}{k}$  is a 'standardized slip' and  $X$  is the corresponding function giving  $H$  in terms of  $H_{\max}$ .

The following terms are significant in the contributions that they make to the tractive force.

- (i)  $c$  and  $\tan \phi$  : the soil strength parameters contribute to  $H$  through their contribution to  $H_{\max}$ .
- (ii)  $A (= b \cdot \ell)$  : the track area contributes to  $H$  through the contribution of the cohesive component of the soil strength to  $H_{\max}$ ; it will be proportional to  $A$  for a purely cohesive soil for which  $\phi = 0$ .
- (iii)  $W (= b \cdot \ell \cdot \gamma)$  : the weight contributes to  $H$  through the contribution of the frictional component of the soil strength to  $H_{\max}$ ; it will be proportional to  $W$  for a frictional soil for which  $c = 0$ .
- (iv)  $\ell$  : the track length contributes to  $H$  through its contribution to track area as explained above. It also contributes as it appears in the slip function in a way that causes an increase in  $X$  as length increases; thus track length has significant effect on  $H$  in addition to its area effect.
- (v)  $k$  : decreasing the horizontal deformation modulus (having a more rigid soil that reaches its maximum shear stress at lesser deformations) has the effect of increasing  $H$  by causing an increase in  $X$ .
- (vi)  $i$  : increasing the slip increases the deformation and the associated shear stress, which has the effect of increasing  $X$  and  $H$ .

The above analysis may be extended to a wheel if it is assumed that the pressure under the wheel is constant. The area of the contact patch may be assumed to be  $0.78 b \ell$ .

#### 4.5 DRAWBAR PULL

The above gives the tractive force - slip relationship for a track or wheel. It is clear that it also gives the basic form to the drawbar pull - slip relationship for the performance of tractors measured in the field where the drawbar pull is what remains of the tractive force after the rolling resistance has been overcome.

Fig. 4.15 shows the comparative performance of the same basic tractor (New Fordson Major) with different wheel equipment, viz, two wheel drive (2WD) , four wheel drive (4WD) and tracks on cultivated (loose) and stubble (firm, rigid) soil (Anon., undated).

From these results it is clear that the following give reduced slip and increased drawbar pull:

- (i) tracks compared to 2WD on cultivated (loose) soil which shows the effect of area and length of contact patch and of weight
- (ii) tracks on stubble (firm) compared to cultivated (loose) soil which shows the effect of soil strength and rigidity (deformation modulus)
- (iii) 4WD compared to 2WD on cultivated (loose) soil which shows the effect of area and weight.

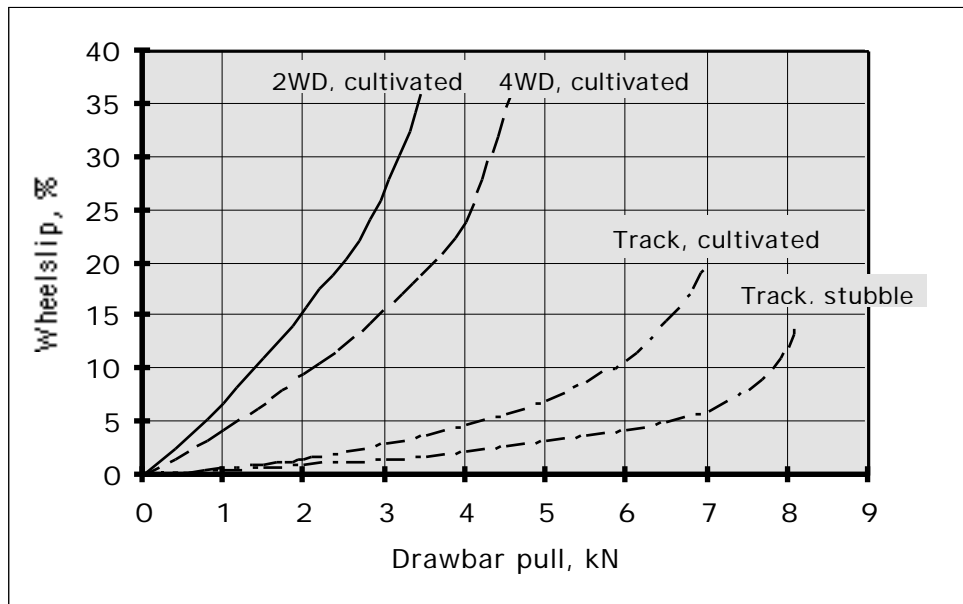


Figure 4.15 Comparative slip - pull performance of two wheel drive, four wheel drive and tracked tractor. Reproduced from Anon. (undated) with permission of Silsoe Research Institute.

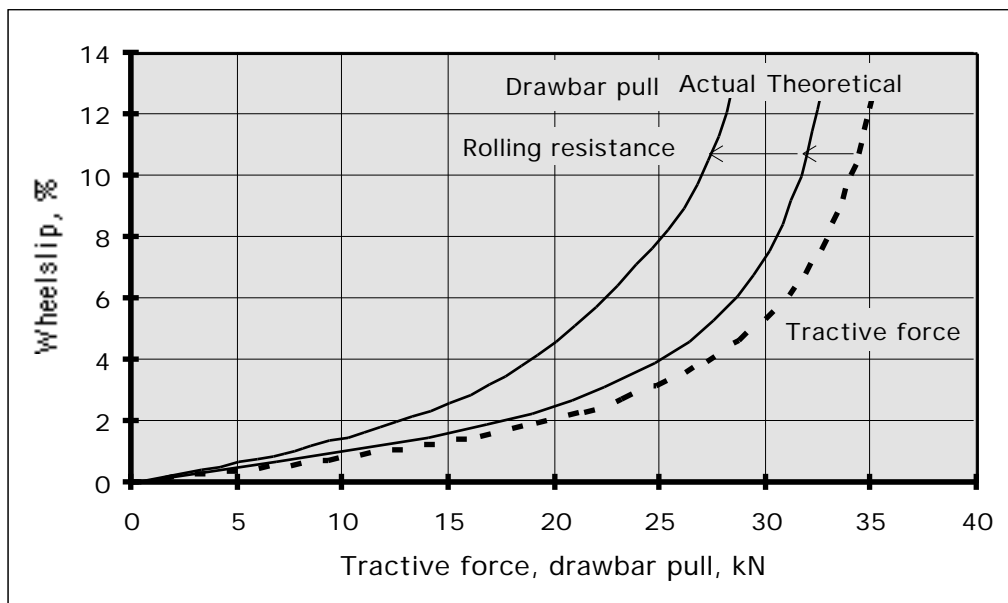


Figure 4.16: Tractive force, actual and predicted drawbar pull for Problem 4.3

As noted above,  $R$  is usually measured or predicted on the assumption of zero slip. An increase in slip associated with an increase in  $H$  will mean that the prediction becomes less accurate because the above assumption will be increasingly invalid.

Hence the prediction of  $P$  using Equation 4.5:

$$P = H - R \quad (4.5)$$

will also become less accurate as shown in Problem 4.3, Figure 4.16.

### Problem 4.3

Figure 4.16 shows a plot of measured wheel slip - drawbar pull data for a small crawler tractor tested on soil. The following data apply:

Tractor:		Soil (assumed):	
Weight, $W$	= 3800 kg	Soil cohesion, $c$	= 15 kPa
Track length, $\ell$	= 1.65 m	Soil angle of friction,	= $30^\circ$
Track width, $b$	= 0.35 m	Soil deformation modulus, $k$	= 0.02 m
Rolling resistance, $R$	= 2.5 kN		

Calculate the theoretical wheel slip - drawbar pull performance and plot it on the graph with the performance from the actual test.

$$H_{\max} = Ac + W \tan$$

$$= 2 \times 0.35 \times 1.65 \times 15 + 3.8 \times 9.8 \times \tan 30 = 38.8 \text{ kN}$$

For appropriately chosen values of  $i$ , calculate  $\frac{i\ell}{k}$  hence  $X$  from Equation 4.20 or read from

Figure 4.14. Calculate  $H$  from Equation 4.19 and  $P$  from Equation 4.5 above. The results are shown plotted in Figure 4.16

## 4.6 DRAWBAR POWER

### 4.6.1 Wheel-slip - drawbar power characteristic

While the wheel-slip - drawbar pull graph above is the main performance characteristic for a track (or wheel) the user is, however, usually more concerned with work rates, ie, drawbar power. The drawbar power - slip results (of the tractor tests shown in Figure 4.15) have been plotted in Figure 4.17 and show that there is an optimum slip that gives a maximum drawbar power. Since all of these tractors had the same engine power, Figure 4.17 also shows (in relation to maximum drawbar power) how significant the soil condition is (track - stubble compared to track - cultivated) and also wheel / track contact area and weight are (track compared to 2WD and 4WD each for cultivated soil).

Figure 4.17 also shows the much greater power obtained from the track (and to a lesser extent the 4WD) due to the larger drawbar pull that can be achieved without excessive slips and the losses in power that are associated with them.

The wheelslip - drawbar power characteristic may be plotted from experimental data as shown in Problem 4.4.

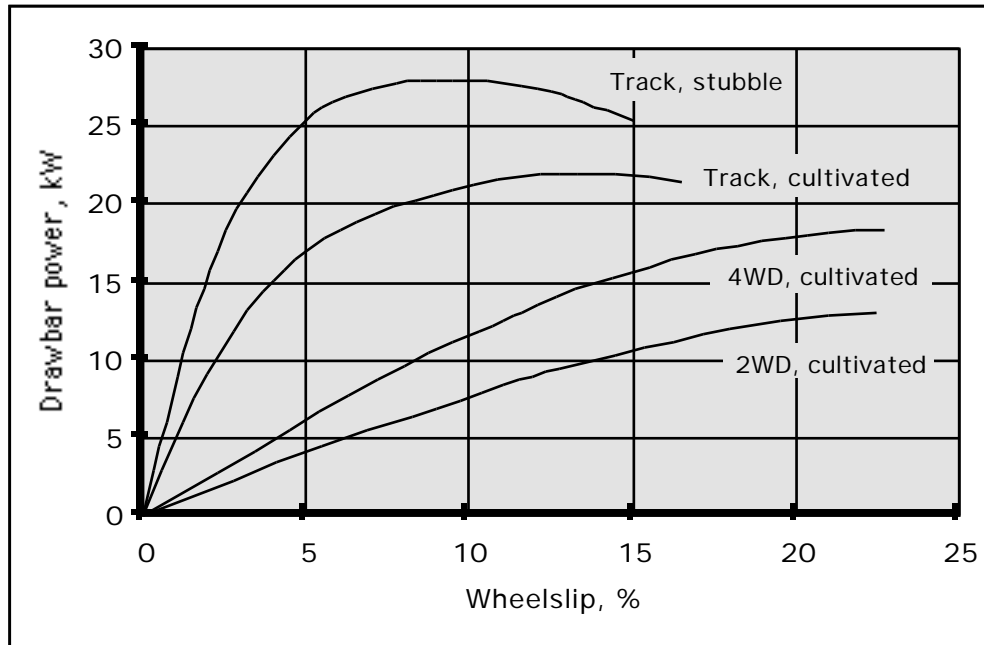


Figure 4.17: Comparative power – slip performance of two-wheel drive, four-wheel drive and tracked tractor. Reproduced with permission of Silsoe Research Institute

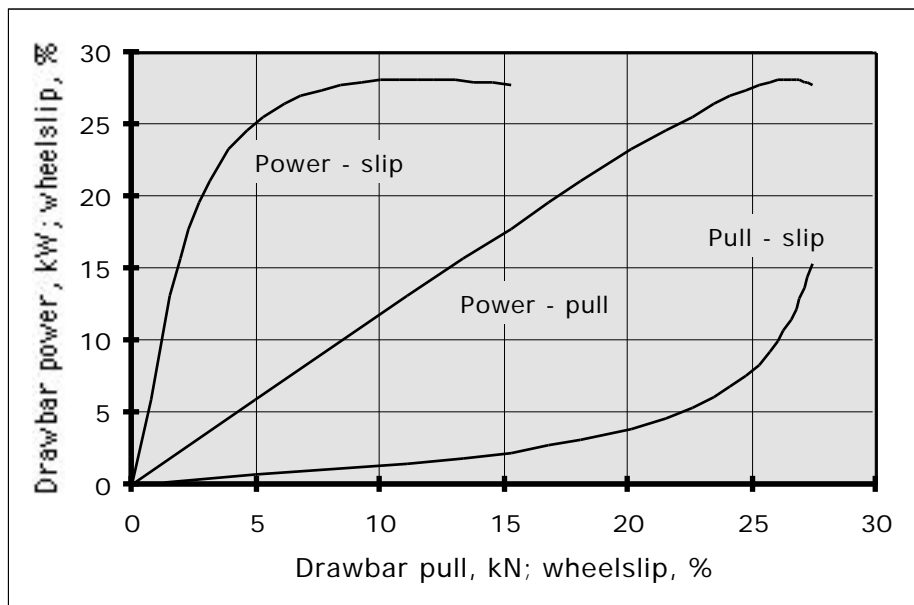


Figure 4.18 Slip- drawbar power - drawbar pull performance for Problem 4.4



**Problem 4.4**

Figure 4.18 shows the track slip - drawbar pull graph for a tracked tractor. The following data apply:

$$\begin{aligned}\text{Diameter of track} &= 0.85 \text{ m} \\ \text{Engine speed} &= 1760 \text{ rpm (assumed constant)} \\ \text{Overall gear ratio} &= \frac{1}{65}\end{aligned}$$

Plot:

- (i) drawbar power versus drawbar pull
- (ii) drawbar power versus track slip, hence
- (iii) determine the conditions for maximum power

$$\text{Linear speed of track } V_o = D N = \frac{3.14 \times 0.85 \times 1760}{65 \times 60} = 1.2 \text{ m/s} \quad (\text{Equation 2.1})$$

Travel speed,  $V = 1.2 (1-i)$  where the slip,  $i$  is read from Figure 4.18 for various drawbar pulls  
Drawbar power,  $Q = P.V$  as plotted in Figure 4.18

The maximum power of 28 kW is achieved at a drawbar pull = 26 kN and track slip = 11 %

**4.6.2 Theoretical prediction of optimum wheel-slip**

The performance of a track may be best characterized by the drawbar pull and slip at maximum drawbar power; this may be predicted as follows (Reece 1967).

From Equations 4.5:

$$\begin{aligned}P &= H - R \\ &= (Ac + W \tan \phi) X - R\end{aligned} \quad (4.21)$$

Drawbar power:

$$Q = P.V$$

But from Equation 2.6

$$\begin{aligned}V &= V_o (1-i) \\ Q &= V_o (1-i) [(Ac + W \tan \phi) X - R]\end{aligned} \quad (4.22)$$

where  $V_o$  = tractor wheel speed.

In order to determine the slip for maximum drawbar power by differentiation, it would be necessary to know  $R$  as a function of slip. This is not available so an alternative is to neglect the influence of  $R$  relative to  $H$  and to calculate slip for maximum tractive power  $Q'$ , ie, obtain a maximum for:

$$\begin{aligned}Q' &= H . V \\ &= V_o(1-i) (Ac + W \tan \phi) X \\ &= V_o (1-i) (Ac + W \tan \phi) \left[1 - \frac{k}{i\ell} + \frac{k}{i\ell} e^{-i\ell/k}\right] \\ &= V_o (Ac + W \tan \phi) \left\{ \left[1 - \frac{k}{i\ell} + \frac{k}{i\ell} e^{-i\ell/k}\right] - \left[i - \frac{k}{\ell} + \frac{k}{\ell} e^{-i\ell/k}\right] \right\}\end{aligned}$$

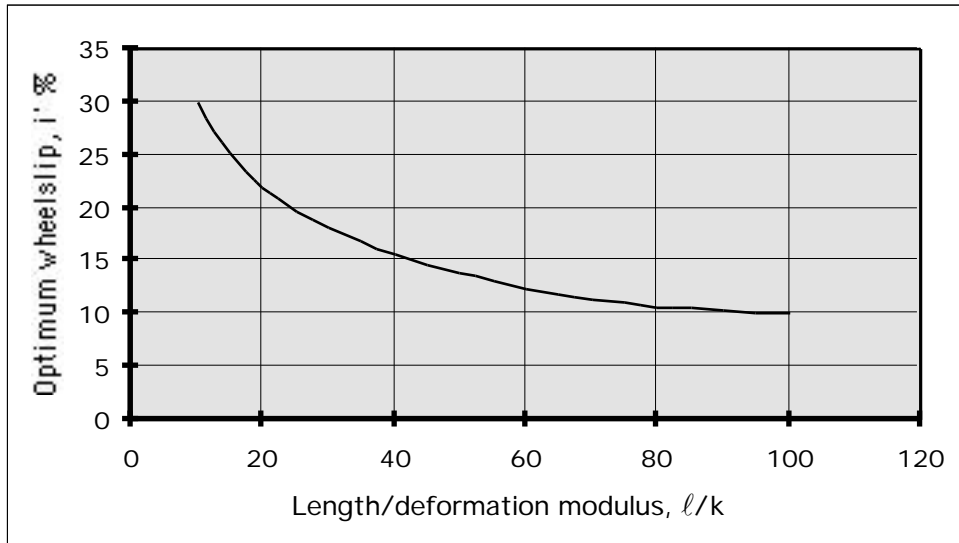


Figure 4.19: Optimum wheelslip as a function of track length/deformation modulus. Reproduced from Reece (1967) with permission of Institution of Agricultural Engineers.

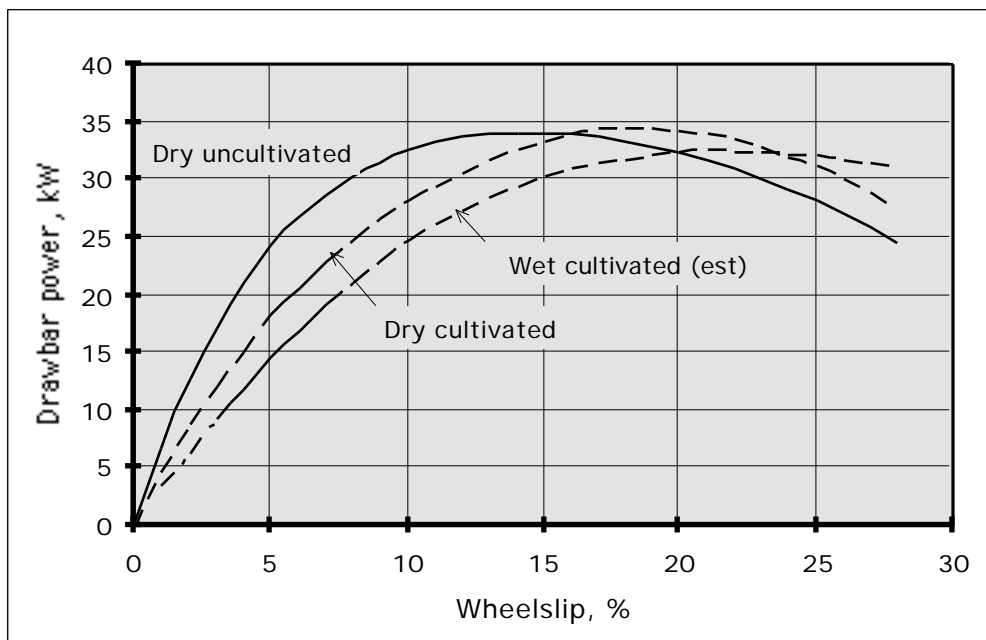


Figure 4.20: Drawbar power - wheelslip performance of a tractor on soil in different conditions. Reproduced from Hutchings (1980) with permission of Department of Natural Resources and Environment (Vic)

Differentiating this with respect to slip gives:

$$\begin{aligned}\frac{dQ'}{di} &= V_o (Ac + W \tan \phi) \left\{ \left[ \frac{k}{i^2 \ell} - \frac{1}{i} e^{-i\ell/k} - \frac{k}{i^2 \ell} e^{-i\ell/k} \right] - [1 - e^{-i\ell/k}] \right\} \\ &= 0 \\ \text{ie, } \frac{k}{i^2 \ell} + e^{-i\ell/k} \left[ 1 - \frac{k}{i^2 \ell} - \frac{1}{i} \right] - 1 &= 0\end{aligned}\quad (4.23)$$

Reece gives the numerical solution to this equation in Figure 4.19 and uses it to give, in terms of the ratio  $\ell/k$ , an approximation to the slip  $i'$ , at which maximum drawbar power is obtained.

This relationship suggests that the slip  $i'$ , at maximum drawbar power (strictly maximum tractive power) decreases:

- (i) as  $k$ , the deformation modulus decreases, ie, the soil becomes more rigid and approaches its maximum shear stress at smaller deformations
- (ii) as  $\ell$ , the length of the contact area, increases

The drawbar power - slip results (of the tractor tests shown in Figure 4.15) which have been plotted in Figure 4.19 confirm this prediction, viz:

- (i) the track reaches maximum drawbar power at smaller slips on rigid stubble than on the loose cultivated soil;
- (ii) the longer track reaches maximum power at a (very much) smaller slip than does the shorter wheel (both on loose soil).

As another example Figure 4.20 shows the graph of drawbar power versus slip for a Deutz 2WD tractor tested on soil in three conditions (Hutchings, 1980). Again the drawbar power is reached at lower slips on the uncultivated (more rigid) soil than on the dry cultivated (loose) soil and both than on the soft, wet, cultivated soil.

The optimum slip ( $i'$ ) obtained from Figure 4.19 can be used, together with an appropriate rolling resistance  $R$ , to calculate the maximum drawbar power.

$$Q_{\max} = V_o (1 - i') [(Ac + W \tan \phi) (1 - \frac{k}{i' \ell} + \frac{k}{i' \ell} e^{-i' \ell/k} - R)] \quad (4.24)$$

The setting up of the tractor to operate at this or other condition is discussed in Chapter 7.

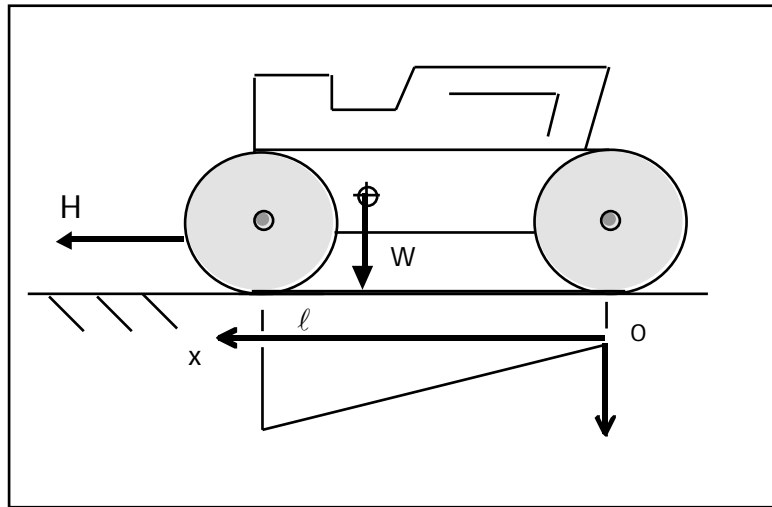


Figure 4.21: Tractor showing normal stress increasing linearly from front to rear as in Problem 4.6. Reproduced from Wills (1963) with permission of Elsevier Science.

#### 4.7 GENERAL PROBLEM

##### Problem 4.5 (Wills 1963)

Develop an expression for the tractive force - slip relationship developed by a track of width  $b$ , length  $\ell$  and total weight  $W$ , operating on a frictional soil, if the normal stress increases linearly from zero at the front to a maximum at the rear as show in Figure 4.21.

Assume  $p = a x$  where  $a$  is a parameter to be determined. The requirement for vertical equilibrium is that:

$$\begin{aligned} W &= \int_0^\ell a b x \, dx \\ &= a b \left[ \frac{x^2}{2} \right]_0^\ell \\ &= a \frac{b \ell^2}{2} \end{aligned}$$

Hence

$$\begin{aligned} a &= \frac{2W}{b \ell^2} \\ &= \frac{2W}{b \ell^2} x \\ s &= \tan^{-1} (1 - e^{-j/k}) \\ &= \frac{2W}{b \ell^2} x \tan^{-1} (1 - e^{-j/k}) \end{aligned}$$

From Section 4.4.4

$$\begin{aligned} H &= \int_0^\ell b s \, dx \\ &= \frac{2W}{\ell^2} \tan^{-1} \int_0^\ell x (1 - e^{-j/k}) \, dx \end{aligned}$$

Substituting  $j = ix$  and integrating gives

$$\begin{aligned} H &= \frac{2W}{\ell^2} \tan^{-1} \left[ \frac{x^2}{2} + \frac{xk}{i} \cdot e^{-ix/k} - \frac{k}{i} \left( -\frac{k}{i} e^{-ix/k} \right) \right]_0^\ell \\ &= W \tan^{-1} \left[ 1 - 2 \left( \frac{k}{i\ell} \right)^2 (1 - e^{-i\ell/k}) - \frac{i\ell}{k} e^{-i\ell/k} \right] \end{aligned}$$

##### Problem 4.6

Plot the tractive force - slip graph for the track in Problem 4.5 using data from Problem 4.3. Compare the answers.

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# **The Mechanics of Tractor - Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

## **CHAPTER 5**

### **TRACTOR PERFORMANCE ON SOFT SOIL - EMPIRICAL**

*Printed from: <http://www.eprints.unimelb.edu.au>*

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 5

### TRACTOR PERFORMANCE ON SOFT SURFACE - EMPIRICAL

#### 5.1 INTRODUCTION

##### 5.1.1 General

In Chapter 3 we considered the experimental evaluation of the 'ideal' performance of the tractor in terms of the engine and the tractor operating in various gears on a firm surface. While this shows the influence of those elements and is valuable for comparative purposes, it is of limited use in showing the performance on soft surfaces or for predictive purposes.

In Chapter 4 we considered the theoretical analysis which is conceptually correct in the way that it calculates the performance of the tractor as determined by the capacity of the surface to generate a reaction. However this approach has two difficulties associated with the measurement of the surface properties.

Firstly, it requires the measurement of six properties of the soil; three (cohesion, angle of internal friction and the deformation modulus) for the prediction of the tractive force and a further three (two sinkage moduli and an exponent) for the prediction of the rolling resistance. These require complex facilities and are likely to be time consuming if representative samples of the properties over an area are to be obtained.

Secondly, in general, agricultural soils are in a structured state in which the bonds between the soil aggregates are intact. Prediction of tractor performance in the field, based on the above properties, requires that the latter be measured with the soil in this undisturbed state. If the soil is disturbed during the sampling process for laboratory determination of the properties (as it is likely to be) the measured values of both cohesion and deformation modulus will be affected.

It is not possible to recreate undisturbed conditions in the laboratory after a soil has been disturbed and hence in-situ methods of measuring undisturbed soil properties have been developed (Baladi, 1987).

##### 5.1.2 Empirical method

The alternative to experimental measurement or to a theoretical analysis that is adopted in many engineering fields is the so-called 'empirical' approach. This is based on a series of experiments that includes the major variables or groups of variables. From these a set of predictive equations is developed, often using techniques of dimensional analysis (Langhaar, 1978).

These equations can replace much experimental work, allow designs to be tried 'on the drawing board' and answer 'what if . . . ?' questions. The designs and the answers are of course only as good as the choice of variables, the experimental data and the fit of the equations that are based on them.

The empirical approach (now frequently termed (computer) modelling) has proved to be useful in many complex engineering problems. It provides a ready and useful means of performance prediction for the tractor but it is not suitable as a basis for understanding the fundamentals of the processes involved. It has mainly been applied to the tractive processes but it may also include the engine and so provide a basis for predicting the performance for the tractor as a whole.

#### 5.2 ENGINE PERFORMANCE MODELLING

Persson (1969) developed an equation for modelling engine performance based on power, speed, swept volume and heat value of the fuel, together with two constants estimated from the test data. However for an engine of given type and swept volume his equation can be reduced to the form given by Huynh and Brown (1981).

$$FC = A Q + B N^2 \quad (5.1)$$

A and B are constants which can be determined if the fuel consumption, engine speed and power are known for two points on the performance characteristic for the engine.

With reference to the performance of the Farmland tractor as shown in Figures 3.2 and 3.3, consider two points on the performance characteristic at maximum governor setting as follows.



		<b>Point 1</b>	<b>Point 2</b>
Engine speed, N, rpm	=	2390	2250
Engine power, Q, kW	=	0	33.5
Fuel Consumption, FC, kg/hr	=	2.8	9.0

Substituting values for point 1 in Equation 5.1, gives

$$B = \frac{FC}{N^2} = \frac{2.8}{2390^2} = 4.9 \times 10^{-7}$$

Substituting values for point 2 in Equation 5.1, gives

$$A = \frac{FC - B N^2}{Q} = \frac{9.0 - 4.9 \times 10^{-7} (2250)^2}{33.5} = 0.19$$

Thus fuel consumption,  $FC = 0.19 Q + 4.9 \times 10^{-7} N^2$  (5.2)

### Problem 5.1

Using Equation 5.2, estimate the fuel consumption for the Farmland tractor for engine power,  $Q = 15$  kW and engine speed,  $N = 1600$  rpm. Compare the answer with the measured value as shown in Figure 3.4.

For  $Q = 15$  kW and  $N = 1600$  rpm, from Equation 5.2,

$$FC = 0.19 \times 15 + 4.9 \times 10^{-7} \times 1600^2 = 2.85 + 1.25 = 4.1 \text{ kg/hr}$$

From the specific fuel consumption lines on Figure 3.4 for  $Q = 15$  kW and  $N = 1600$ ,  $SFC = 250$  g/kWhr.

Measured  $FC = 250 \times 15 = 3.8$  kg/hr

The predicted value is within 8% of the measured value which is about the accuracy that can be expected with the empirical approach.

## 5.3 TRACTIVE PERFORMANCE MODELLING

### 5.3.1 Parameters

In the empirical prediction of tractive performance, only one soil parameter is measured for the prediction of both tractive force and rolling resistance. This parameter, known as the 'cone index', is not dependent on the measurement of deformation or sinkage as is required in the determination of the respective moduli in the theoretical approach.

Its measurement, being so simple, allows a rapid survey of the area of interest and incidentally reveals the great variability that frequently exists in both time and place, particularly due to the variation in soil texture and the effect of moisture content.

The development of the algorithms that constitute the tractive model requires an extensive series of measurements of cone index and corresponding tractor performance as reported by Frietag (1965), Wismer and Luth (1974), Gee-Clough et al (1978) and Parkhill (1986).

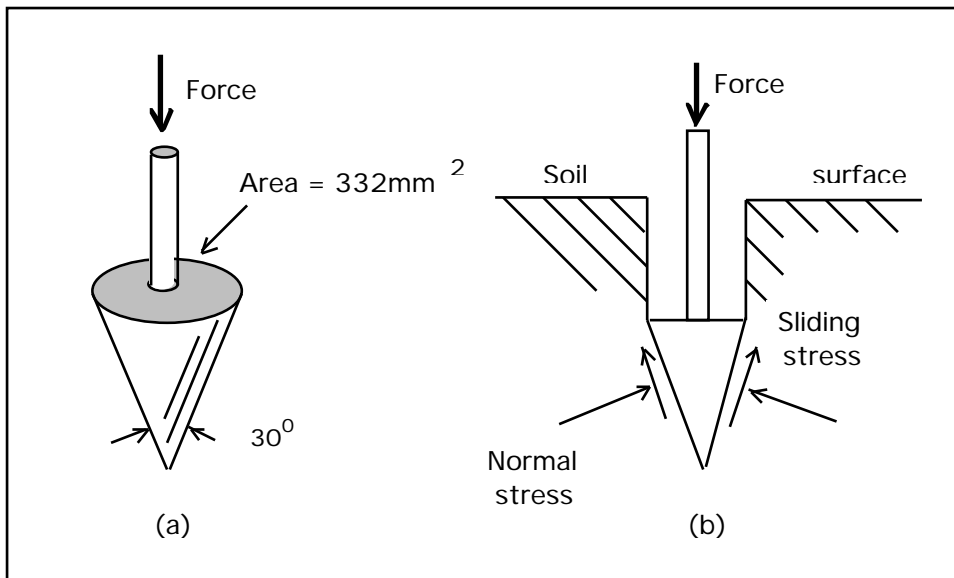


Figure 5.1: Cone penetrometer for measurement of soil parameter

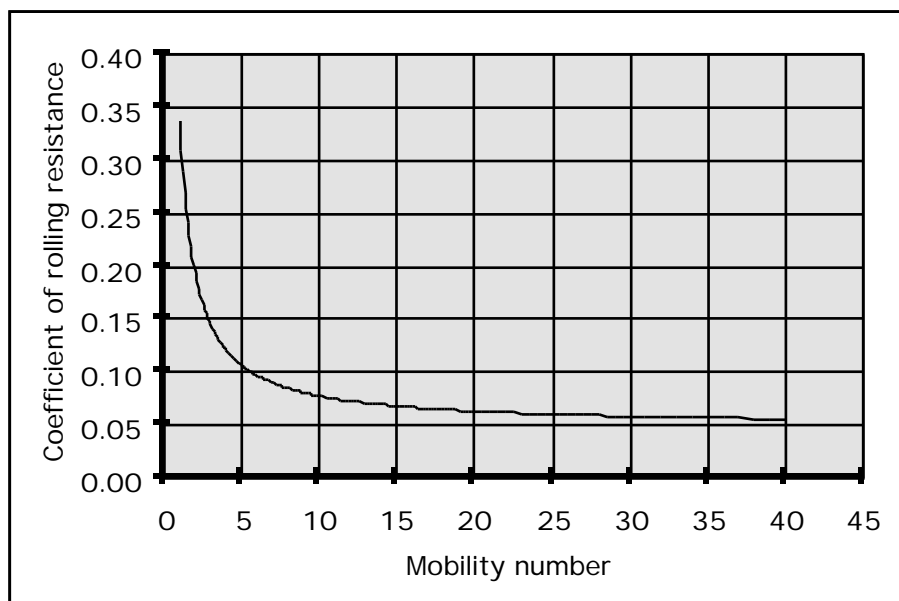


Figure 5.2: Variation of coefficient of rolling resistance with mobility number.

(a) Cone index

The soil parameter for empirical prediction of tractive performance is based on the force (kN) to push a circular cone (base area = 0.5 in<sup>2</sup>; 322 mm<sup>2</sup>) shown in Figure 5.1(a) into the soil at a constant speed of 72 in/min (30 mm/sec) (ASAE, 1998).

The parameter, termed the cone index is given by,

$$CI = \frac{\text{Force on cone}}{\text{Base area of the cone}} \quad \text{kPa} \quad (5.3)$$

The passage of the cone into the soil is resisted by the normal and soil - metal resistance forces as suggested in Figure 5.1(b). These in turn will depend on the strength and compressibility of the soil and soil / metal sliding characteristics of the cone surface all of which will depend on the soil texture, moisture content, etc.

The cone index does not therefore represent a soil 'property' as such but a complex and ill defined parameter or measure of soil 'strength' and deformability; it is assumed to be a correlate for tractive force and rolling resistance.

Typical values of cone index are given by Dwyer (1976) as shown in Table 5.1.

Surface Condition	Cone Index, kPa
Dry grassland	1500
Dry stubble	1000
Wet stubble	500
Dry loose soil	400
Wet loose soil	200

Table 5.1: Soil cone index for various surface conditions. Reproduced from Dwyer et al (1976) , with permission of Silsoe Research Institute.

The interesting aspect of this table is that it is based on:

- (i) soil 'condition' as represented by the terms loose, stubble (implying moderately firm) and grassland (implying firm )
- (ii) moisture condition as implied by the terms dry and wet

Soil texture is not specified because the above variables are seen to have the most significant effect on cone index.

(b) Mobility number

The early work on the empirical prediction of the performance of wheels on soft surfaces was carried out by Fretag (1965) in a military context. In this approach, dimensional analysis was used to effectively reduce the number of variables and so simplify the prediction equations.

This was applied to wheels on agricultural soils as reported by Wismer and Luth (1974) also Dwyer et al (1976). The latter authors used the cone index to calculate a dimensionless, tyre mobility number:

$$M = \frac{CI \cdot b \cdot d}{W} \sqrt{\frac{d}{h \cdot d + 0.5b}} \quad (5.4)$$

where

- M = mobility number
- CI = cone index, kPa
- W = weight on tyre, kN
- b, d, h = tyre width, tyre diameter, tyre section height, m
- = tyre deflection under weight W, m

They also established the empirical relationships (for soft surface conditions) between tyre mobility number and the performance parameters discussed in the following sections.

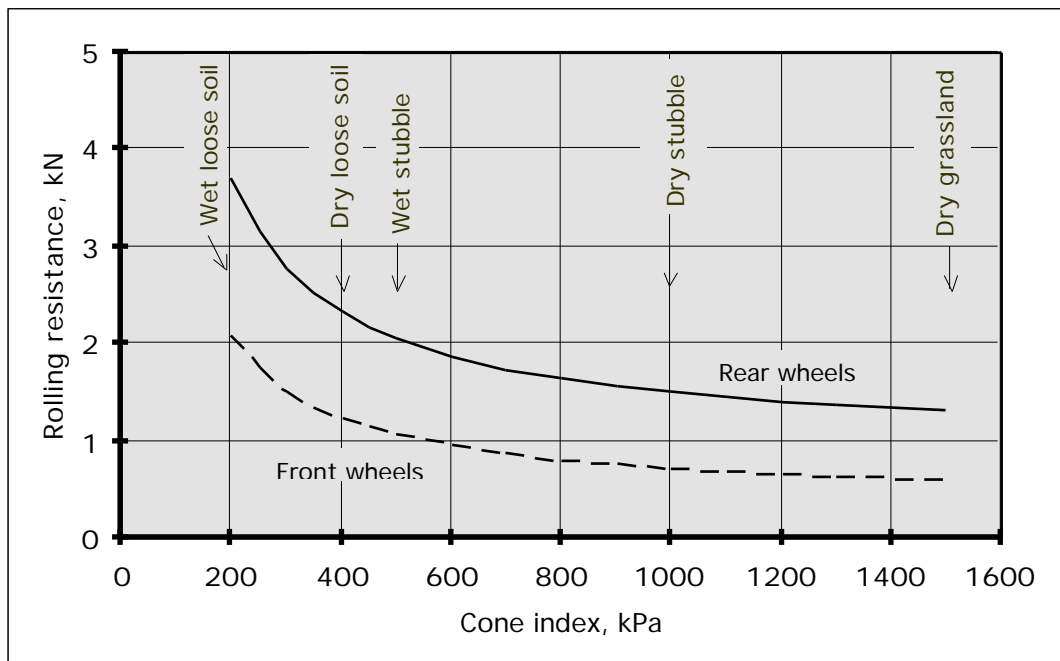


Figure 5.3: Effect of surface / soil condition on rolling resistance of wheels of the Farmland tractor.

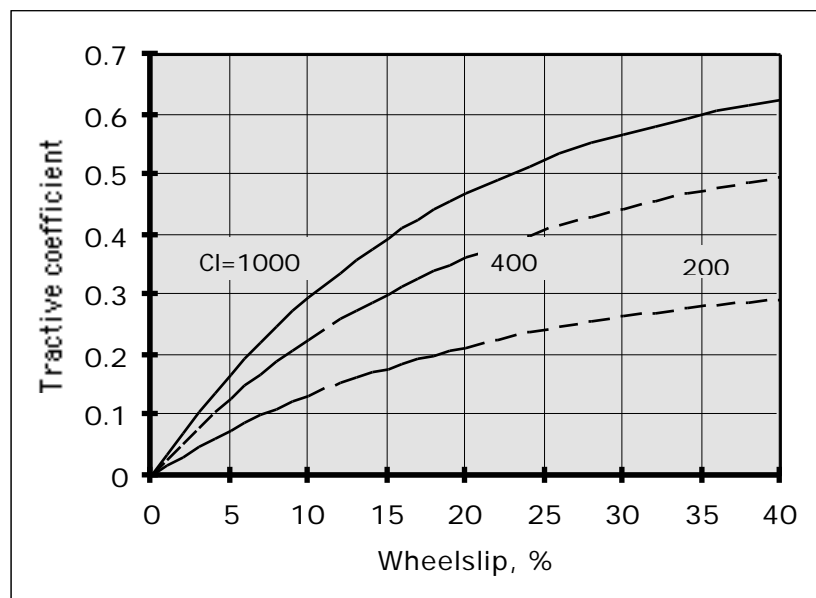


Figure 5.4: Variation of tractive coefficient with wheelslip for three surface conditions for the drive wheels of the Farmland tractor.

### 5.3.2 Prediction of performance measures

In the following sections the various measures of performance have been plotted for the range of cone index values over which the predictive equations apply. These range from 200 for loose wet soil to 1500 for dry firm grassland (Table 5.1). They are based on the mobility number using the static weights on the wheels in Equation 5.4; a second iteration based on the dynamic weight gave no significant difference in the results.

#### (a) Rolling resistance

The following equation was fitted to the rolling resistance data by Gee-Clough et al (1978).

$$= 0.049 + \frac{0.287}{M} \quad (5.5)$$

This is shown plotted in Figure 5.2 and shows how the coefficient increases significantly for small values of M (Equation 5.4) and hence for:

- |       |                            |                       |
|-------|----------------------------|-----------------------|
| (i)   | small values of CI         | - soft surface        |
| (ii)  | small values of d          | - small diameter tyre |
| (iii) | small values of b          | - narrow tyre         |
| (iv)  | small values of $\sqrt{h}$ | - stiff tyre          |
| (v)   | large values of W          | - large weight        |

It also shows how the rolling resistance coefficient approaches a value of about 0.05 for firm surfaces (large values of M and hence of CI, d and b etc).

The rolling resistance is then calculated as the product of this coefficient and the dynamic weight on the tyre as discussed in Section 4.3.3

$$R = V \quad (5.6)$$

Figure 5.3 shows by way of example the rolling resistance for the two rear and two front wheels of the Farmland tractor when operating without drawbar pull on surfaces with a range of cone index values. The values of rolling resistance also represent the power loss in kW for each metre / second of travel speed.

Empirical data for rolling resistance of various tyres carrying various weights are given in Dwyer et al (1976).

It is interesting to note that here, as in Chapter 4, no account is taken of the effect of wheelslip on rolling resistance.

#### (b) Tractive coefficient<sup>1</sup>

The following equations, which were fitted to the traction data by Gee-Clough et al (1978) are equivalent to Equation 4.21.

$$= \max (1 - e^{-ki}) \quad (5.7)$$

$$\text{where } \max = 0.796 - \frac{0.92}{M} \quad (5.8)$$

$$k = 4.838 + 0.061 M \quad (5.9)$$

Figure 5.4 shows the variation in this tractive coefficient with wheelslip calculated for the Farmland tractor from Equations 5.7 to 5.9.

<sup>1</sup> In the literature this tractive coefficient (represented here as  $\mu$ ) is based on the 'net tractive effort' or the pull generated by the driving wheels, ie, the tractive force less the rolling resistance of those wheels as defined in Equation 4.5. The drawbar pull for the tractor requires the subtraction of the rolling resistance of the front wheels as in Equation 5.10

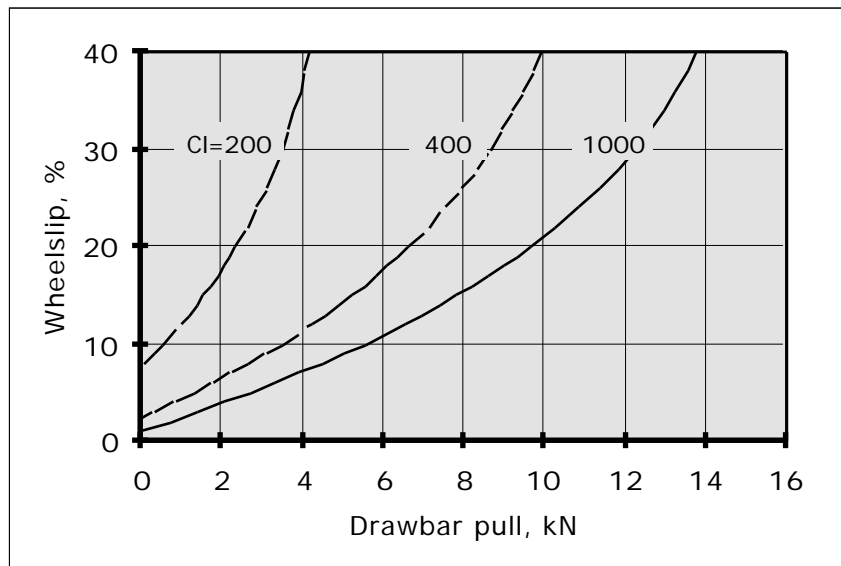


Figure 5.5: Variation in wheelslip with drawbar pull for the Farmland tractor on three surface conditions

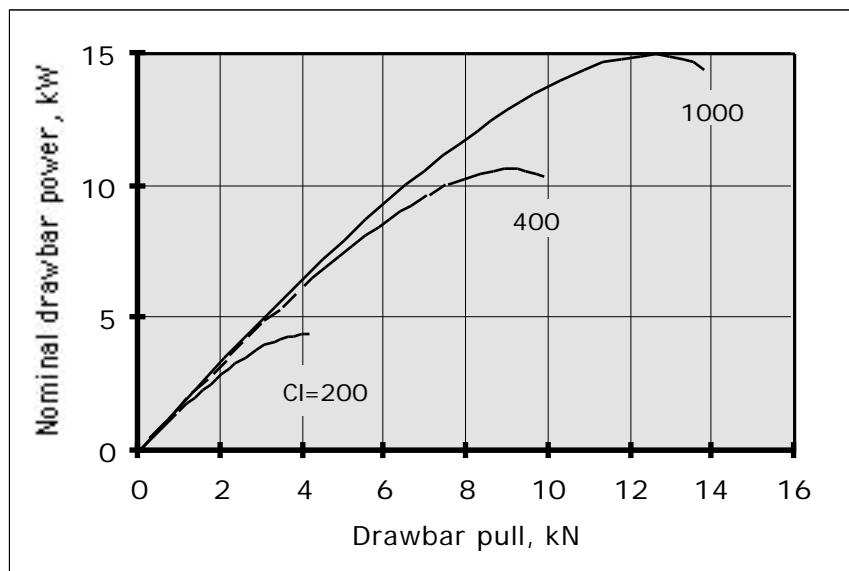


Figure 5.6 Variation in draught power with drawbar pull for the Farmland tractor operating on three surface conditions

(c) Drawbar pull

The drawbar pull for the tractor is then calculated from the tractive coefficient for various wheel slips less the rolling resistance of the front wheels.

$$P = V_r - V_f f \quad (5.10)$$

It is shown in Chapter 6 that the dynamic weight on the front wheels ( $V_f$ ) and rear wheels, ( $V_r$ ) are a function of  $P$ , the corresponding static weights  $W_f$ ,  $W_r$  and the dimensions of the tractor. Thus

$$V_r = W_r + P \frac{y'}{x}$$

$$V_f = W_f - P \frac{y'}{x}$$

Substituting in Equation 5.10 gives

$$P = \frac{W_r - fW_f}{1 - \frac{y'}{x} (1 + f)} \quad (5.11)$$

Figure 5.5 shows the variation in drawbar pull with wheel slip for the Farmland tractor for three surface conditions.

(d) Drawbar power

The drawbar power is then calculated from the drawbar pull and the travel speed as in Equation 2.6.

$$\begin{aligned} \text{DB power} &= P V \\ &= P V_o (1-i) \\ &= P \frac{D N_e}{q} (1-i) \end{aligned} \quad (5.12)$$

Figures 5.6 and 5.7 show the variation in nominal drawbar power with drawbar pull and wheel slip for the Farmland tractor in 5th gear and an assumed constant engine speed of 2250 rpm. Again the performance is shown for three surface conditions.

It will be noticed that the maximum drawbar power and the wheel slip at which it occurs are both dependent significantly on the surface condition.

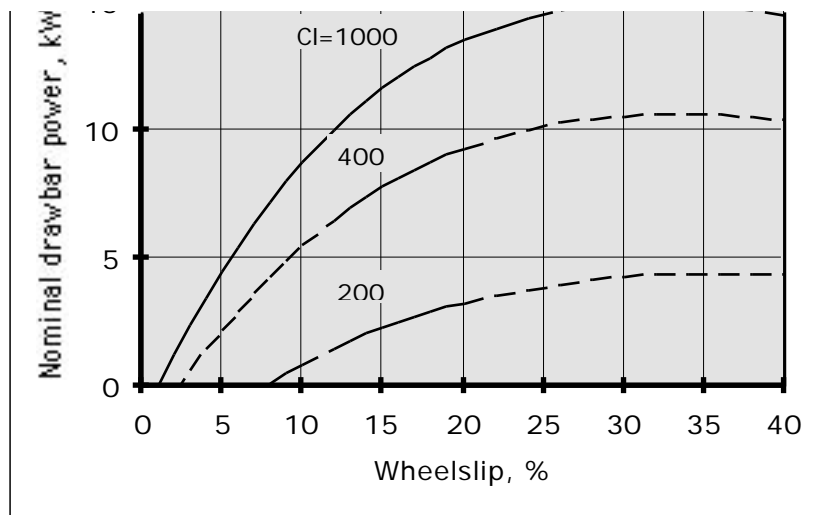


Figure 5.7: Variation of drawbar power with wheelslip for the Farmland tractor operating on three surface conditions

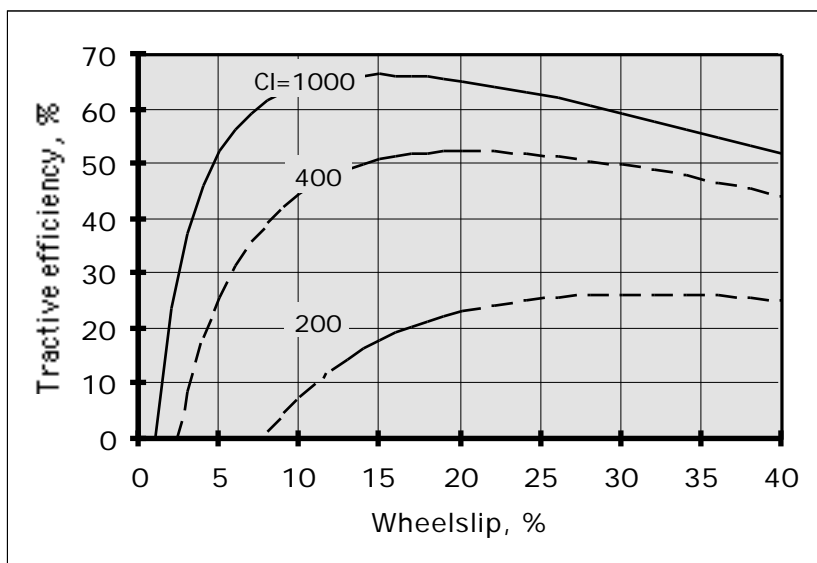


Figure 5.8: Variation of tractive efficiency with wheelslip for the Farmland tractor operating on three surface conditions.



(e) Tractive efficiency

The tractive efficiency is based on an equation of the form given in Equation 2.10.

$$t = \frac{P}{P+R} \quad (1-i)$$

For the tractor as a whole this includes the rolling resistance of the front wheels. Substituting for P and R from Equations 5.6 and 5.10, respectively gives:

$$t = \frac{V_r - V_{f_f}}{(V_r - V_{f_f}) + V_{f_f} + V_{r_r}} \quad (1-i)$$

$$= \frac{P}{V_r + R} \quad (1-i) \quad (5.13)$$

Figure 5.8 shows the variation in tractive efficiency with wheelslip for the Farmland tractor for the three surface conditions. This will be the same for all gears (speeds) because Equation 2.10 is independent of speed.

Again it will be noticed that:

- (i) the maximum tractive efficiency and the wheelslip at which it occurs both depend significantly on the surface condition.
- (ii) the wheelslip at maximum tractive efficiency is much less than that at maximum drawbar power.

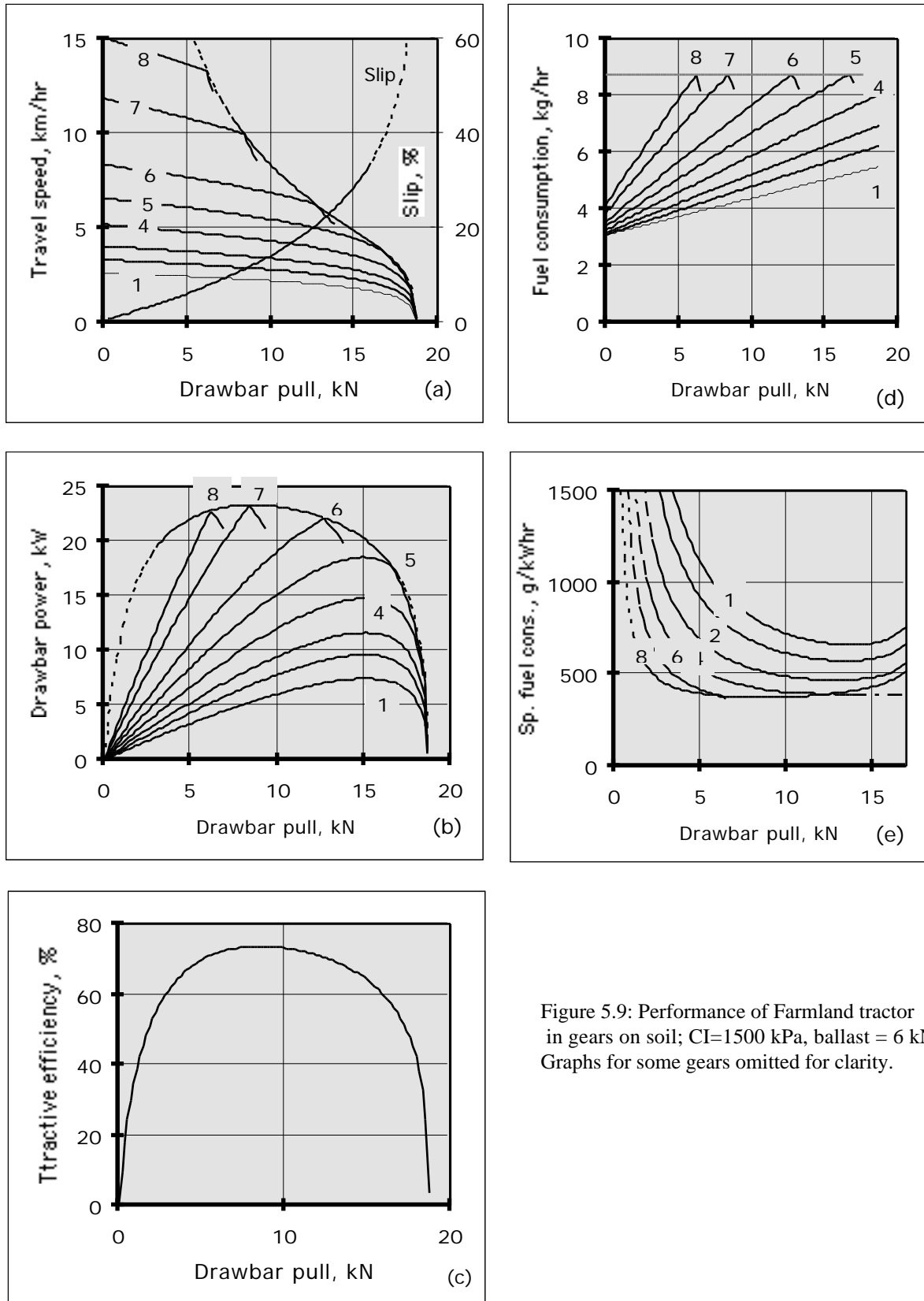


Figure 5.9: Performance of Farmland tractor in gears on soil; CI=1500 kPa, ballast = 6 kN. Graphs for some gears omitted for clarity.

## 5.4 TRACTOR DRAWBAR PERFORMANCE<sup>2</sup>

When the empirical models of the engine and the tractive process are combined we obtain a set of graphs representing the drawbar performance of the tractor as a whole. These may be plotted in various ways to illustrate aspects of the performance that are of interest; in the following, the various measures of performance are plotted against drawbar pull as the independent variable.

### 5.4.1 Performance in various gears

Figure 5.9 shows the results of an analysis of the performance of the Farmland tractor at maximum governor setting with maximum ballast (6kN) on firm grassland (CI = 1500 kPa). Where applicable, the corresponding envelopes of performance, as discussed in Section 2.2.5, are also included. The graphs have been truncated in the full fuel range for clarity. For the cone index and weight values considered, three of the gears are limited by engine torque and five are limited by wheelslip.

Figures 5.9 (a) - (c) show the graphs of travel speed, drawbar power and tractive efficiency versus drawbar pull. There is only a single graph for wheelslip and tractive efficiency because it is assumed that the travel speed and power losses due to wheelslip and rolling resistance are independent of speed and hence gear.

Figure 5.9 (d) and (e) shows the graphs of fuel consumption and specific fuel consumption versus drawbar pull.

The shape of these graphs is consistent with that which would be expected on the basis of the simple theoretical analysis (for torque limited gears) given in Chapter 2 and the experimental results given for all gears when the tractor is tested on a firm surface as given in Chapter 3.

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<sup>2</sup> The assistance of Mr. G. Parkhill in providing the data and algorithms for this section is gratefully acknowledged.

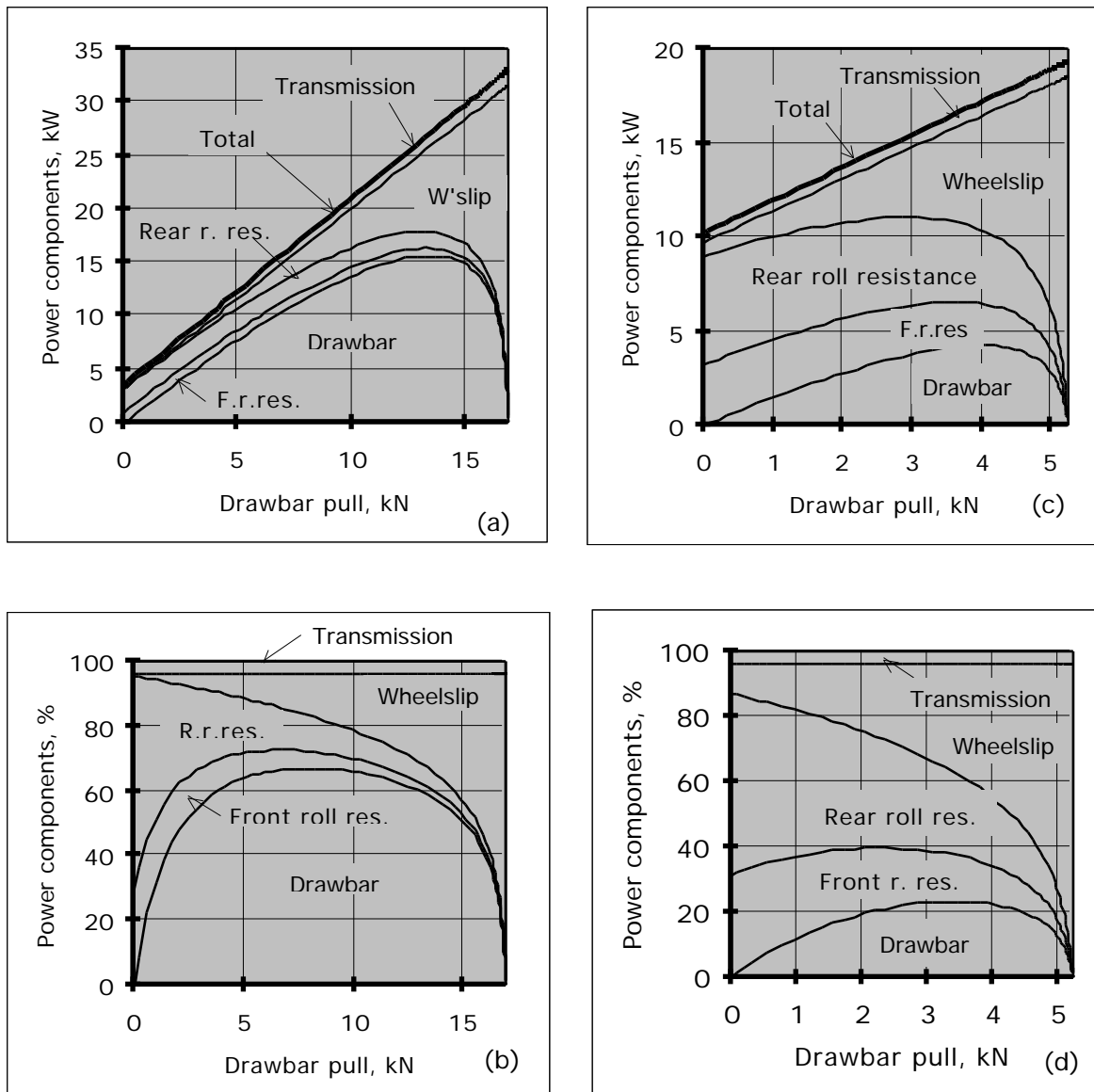


Figure 5.10: Power distribution to drawbar and losses for Farmalnd tractor in 5th gear with 6kN ballast; (a) and (b) for soil, CI = 1500 kPa; (c) and (d) for soil, CI=200 kPa

### 5.4.2 Distribution of power components

Another interesting way to illustrate the performance of the tractor is to calculate the distribution of the components of the total engine power and plot them in absolute and percentage terms. Figure 5.10 (a) and (b) shows this for the Farmland tractor at maximum governor setting with maximum ballast (6 kN) operating in 5th gear on a surface with  $CI = 1500 \text{ kPa}$  (firm grassland). Power losses in the transmission were assumed to be 4% of the total power being transmitted.

Figure 5.10 (c) and (d) shows the distribution for the tractor in the same condition but with soft, wet surface for which  $CI = 200 \text{ kPa}$ . For this gear on both surfaces, the tractor performance is limited by wheelslip.

- (i) Power losses due to wheelslip, which arise from the relative motion of the wheel and ground surface, increase as the drawbar pull increases from the defined zero value at zero drawbar pull to 96% for the maximum sustained pull when the wheelslip is 100%.
- (ii) In terms of the empirical model, the rolling resistance force for both front and rear wheels is assumed to be constant (neglecting the effect of weight transfer); no account is taken of the effect of wheel sinkage due to wheelslip on the rolling resistance of the rear wheels. These power losses therefore decrease as the travel speed decreases due to increased wheelslip. They are a large percentage of the total losses at small drawbar pulls particularly for soft surface.
- (iii) After losses are considered, the remaining power appears at the drawbar. It reaches a maximum when the increase in drawbar power due to increased drawbar pull just balances the increase in power losses mainly due to the increase in wheelslip. The drawbar power, expressed as a % is, in effect, the tractive efficiency; this reaches a maximum at a lower drawbar pull and wheelslip than does the absolute value of drawbar power.

The greater drawbar pull, the smaller power losses and the increased drawbar power that the tractor develops on the firm surface (a) and (b), compared to that on the soft surface (c) and (d), can be seen.

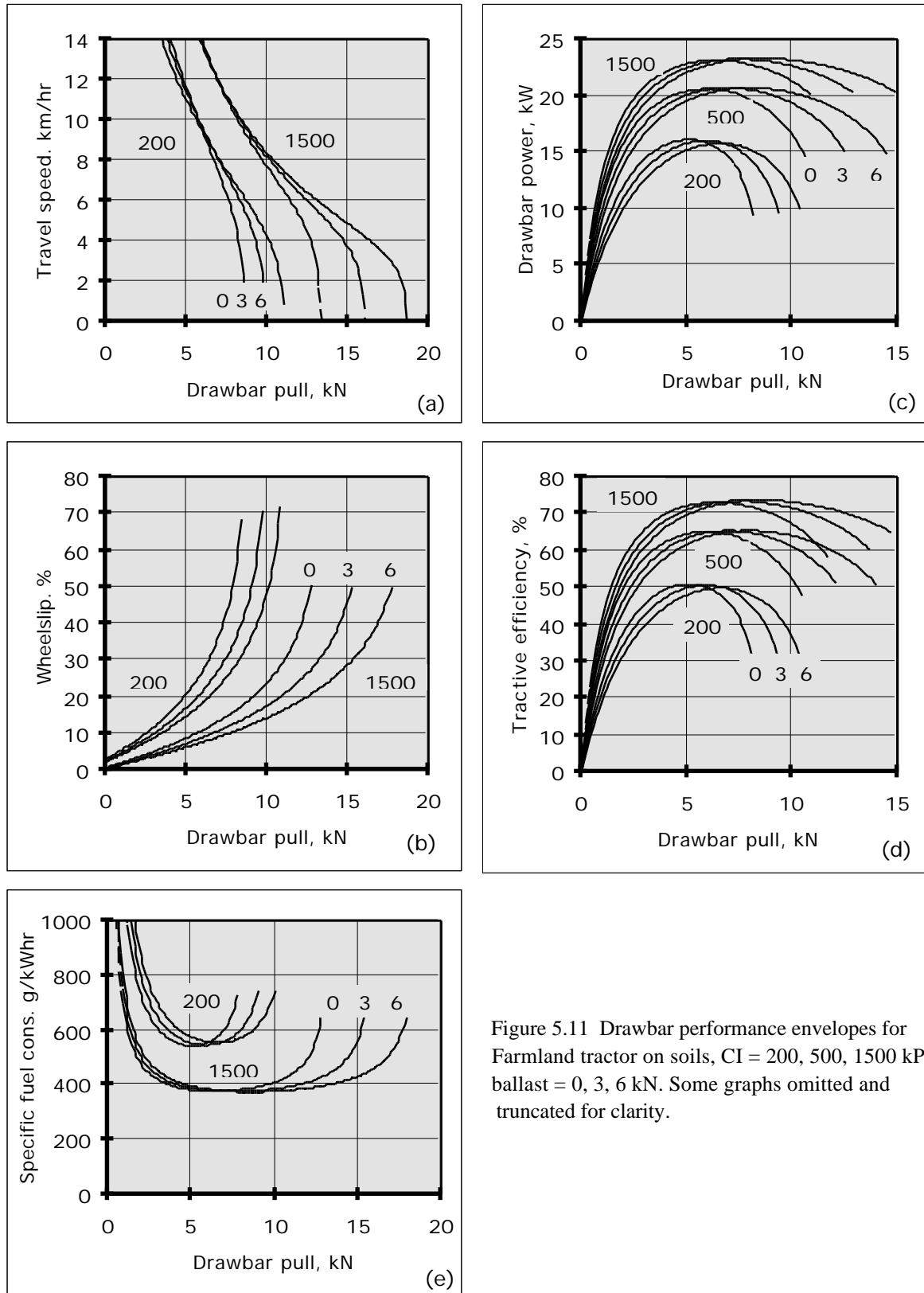


Figure 5.11 Drawbar performance envelopes for Farmland tractor on soils, CI = 200, 500, 1500 kPa; ballast = 0, 3, 6 kN. Some graphs omitted and truncated for clarity.

### 5.4.3 Effect of surface and weight

The graphs in Figure 5.9 show in detail the performance of the tractor in various gears and the envelopes within which the tractor works. The influence of weight and surface condition on performance can be adequately shown by omitting the detailed performance in the gears and considering only the performance envelopes.

Figure 5.11 shows the graphs of travel speed, wheelslip, drawbar power, tractive efficiency, drawbar specific fuel consumption versus drawbar pull. Three values of cone index (CI = 200, 500, 1500 kPa) and 3 levels of added weight (0, 3 and 6 kN). Some of the graphs have been truncated and others at 500 kPa have been omitted for clarity.

#### (a) Drawbar pull

- (i) Surface condition and, to a lesser extent, weight have a significant effect on the maximum drawbar pull.

#### (b) Travel speed and wheelslip

- (i) Surface condition has a significant effect on travel speed and wheelslip at all drawbar pulls.
- (ii) Weight has a small effect on travel speed except at high drawbar pulls. Its effect on wheelslip is significant at all drawbar pulls.

#### (c) Drawbar power and tractive efficiency

- (i) Surface condition has a significant effect on maximum power and maximum efficiency.
- (ii) Weight has:
  - \* a negative effect on maximum power and maximum efficiency for all surface conditions in the higher gears and lower wheelslips where rolling resistance losses predominate.
  - \* a positive effect on maximum power and maximum efficiency for all surface conditions in the lower gears and higher wheelslips where these losses predominate.

These effects are illustrated by the fact that:

- \* for gears giving maximum power and efficiency at drawbar pulls less than about 7 kN, ie higher gears, adding weight decreases the maximum power and maximum efficiency.
- \* for gears giving maximum power and efficiency at drawbar pulls greater than about 7 kN, ie lower gears, adding weight increases the maximum power and maximum efficiency.

- (iii) Surface condition and weight influence the drawbar pull at which maximum power and maximum efficiency occur.

#### (d) Drawbar specific fuel consumption

- (i) Cone index has a significant positive effect (reducing SFC), particularly at high drawbar pulls.
- (ii) Weight has a
  - \* a negative effect (increasing SFC) for all surface conditions in the higher gears and lower drawbar pulls.
  - \* a positive effect (decreasing SFC) for all surface conditions in the lower gears and higher drawbar pulls.

These effects are associated with the performance described in (c) (ii) above.

- (iii) Surface condition and weight influence the drawbar pull at which minimum SFC occurs.

It will be seen from the above that surface condition has the major effect on tractor performance. Optimum performance will be achieved when the tractor is set up with tyres of a size and with a weight that minimizes the losses from both rolling resistance and wheelslip.

The other factor which is not shown by the above is the need to avoid excessive soil compaction. If additional weight is required for tractive purposes, fitting larger tyres, which allows a greater weight to be carried without excessive surface pressure, will usually be desirable.

## 5.5 CONCLUSION

As illustrated in the above examples, empirical modeling provides a powerful analytical tool to investigate the relationships between the tractor parameters and the performance variables. Further experimental work and development of the models will improve their capabilities and provide the user with further assistance in setting up the tractor. This requires consideration, not only by the appropriate tractor parameters but also the choice of implement size to give the appropriate drawbar pull. These matters are discussed in Chapter 7.

Performance models which use data in real time from an operating tractor and a local or global positioning system are now available (Yule, et al, 1999). These will allow the operator and / or the control system to 'learn' from its previous 'experience' and so develop strategies to achieve optimum performance under varying conditions in the field.

### Problem 5.2

Determine the model parameters for a local tractor and plot its performance as described above.

Repeat the analysis for the tractor with:

- (i) smaller tyres
- (ii) larger tyres

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# **The Mechanics of Tractor – Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

## **CHAPTER 6**

### **HITCHING AND MECHANICS OF THE TRACTOR CHASSIS**

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 6

### HITCHING AND MECHANICS OF THE TRACTOR CHASSIS

#### 6.1 INTRODUCTION

It was shown in Chapters 4 and 5 how the weight on the wheels of a tractor determines its tractive force and rolling resistance, hence its drawbar pull and tractive efficiency.

This weight depends on:

- (i) the static forces, viz,
  - \* the weight of the tractor
  - \* that part of the implement weight (if any) that is carried by the tractor
- (ii) the effect on the tractor of the dynamic forces arising from the action of the implement, viz,
  - \* draught (horizontal) force(s)
  - \* vertical force(s)

In designing and using the tractor - implement system, it is desirable to take advantage of all these forces to increase (and control) the weight on the tractor wheels while still ensuring the satisfactory performance of the tractor and the implement. For a given optimum weight on the wheels, the more that is provided by the dynamic effects, the less that has to be provided by the static weight. The three-point linkage system introduced by Ferguson, which made significant use of the dynamic forces on the implement to provide weight on the driving wheels, allowed the introduction of a very light tractor. This feature is now used on most small to medium sized tractors.

Before considering the mechanics of the tractor chassis we need to review the methods of hitching (attaching) implements to the tractor as these have a significant influence on how the implement forces determine the dynamic weight on the tractor wheels. The following gives a brief review of those aspects of implement hitching that are relevant to the performance of the tractor. Other details of the various systems may be found in the references at the end of this Chapter.

#### 6.2 IMPLEMENT HITCHING

##### 6.2.1 Introduction

The hitching of implements and the mechanics of the chassis may be studied by considering two perpendicular planes:

- (i) the vertical longitudinal plane down the centre line of the tractor in which we consider the symmetrical forces such as the weight, the wheel reactions and the direct effect of the implement forces.
- (ii) the horizontal plane where the moment effect of the implement forces which are not symmetrical (eg, unsymmetrical or off-set implements and all draft forces in turning) will affect the attitude and steering of the tractor. These influence the operation of the tractor but are not relevant to the normal (straight ahead) performance of the tractor; they will not be considered further in this book..

The hitching of implements to tractors may be made in various ways and places. For this purpose the tractor has one or more standard attachment locations at the rear and for some tractors at the front, in the form of:

- (i) linkages for 'adjustable' attachment; adjustment in the vertical plane is usually made by means of an in-built hydraulic (hydro-static) pump driven by the tractor engine.
- (ii) drawbars for 'fixed' attachment; adjustment is made manually or with 'external' or 'remote' hydraulic cylinders supplied with oil from the in-built hydraulic pump in the tractor.

The standard hitching systems may be classified as follows.

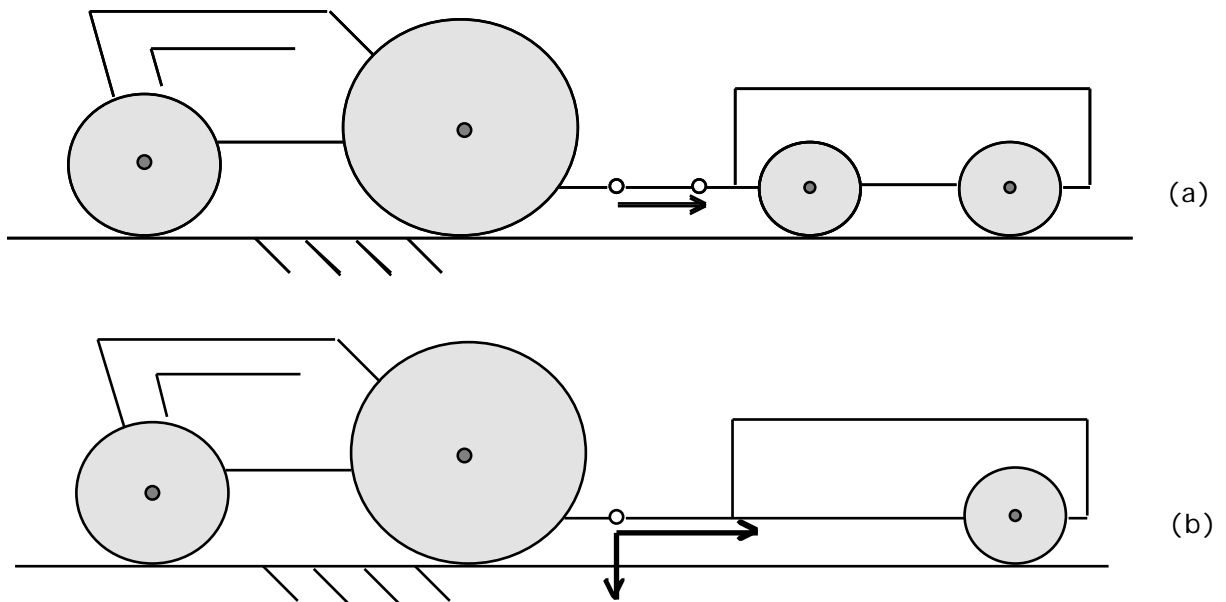


Figure 6.1: Trailed (one point) implement hitches (a) without and (b) with vertical force.

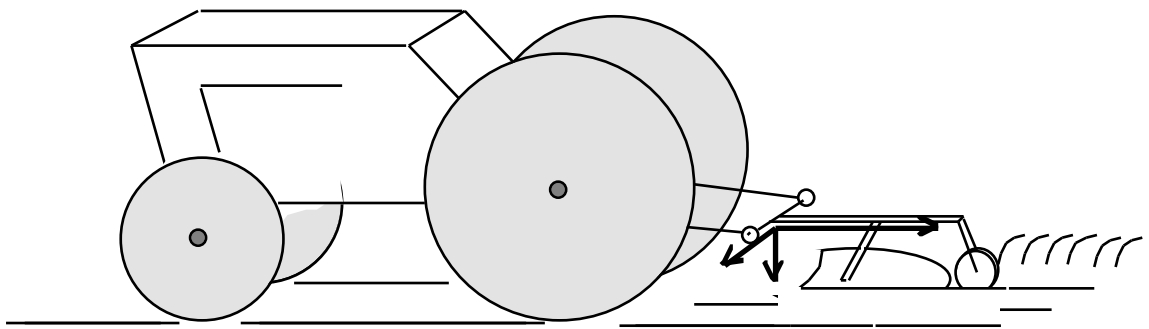


Figure 6.2: Semi-mounted hitch where the front of implement is carried on a horizontal pivot.

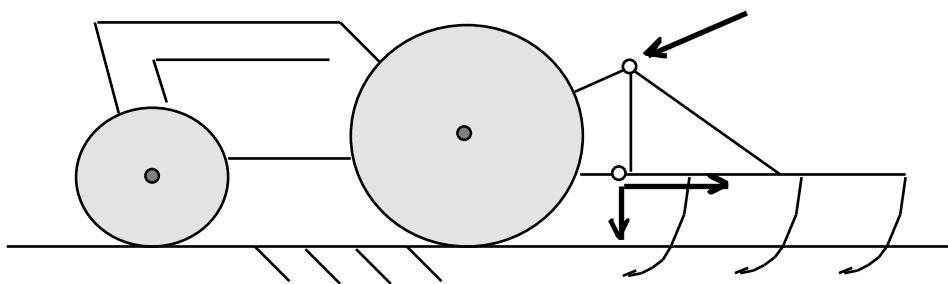


Figure 6.3: Fully mounted, rear three-point linkage hitch.

### 6.2.2 Hitching systems

#### (a) Trailed - one point hitch

Here the implement is attached to the tractor at one (drawbar) hitch point. This represents the simplest arrangement, but it provides a minimum in the way of implement control and weight transfer. The implement, which is usually carried on wheels (for support and / or depth control), is free to move in both the horizontal and vertical planes as it follows the varying ground surface.

Two common arrangements can be identified.

- (i) where the implement is fully carried on its wheels and its drawbar is pivoted at both ends; the implement force is essentially horizontal, Figure 6.1 (a).
- (ii) where the front of the implement (such as in an unbalanced trailer or similar two-wheeled implement) is carried on the tractor drawbar and the rear on a wheel or wheels, Figure 6.1(b). There is usually a significant static vertical component in the implement attachment force and hence the weight transfer from implement to tractor rear wheels is greater than in (i) above.

The traile hitch is least effective in terms of both weight transfer and implement control when compared with other systems (see Section 6.4.3). The former weakness has been overcome by the development of a weight transfer hitch for traile implements in which part of the weight of the implement and / or the downward soil forces are supported by the tractor rear wheels. This system is considered in Section 6.4.4(c).

#### (b) Semi-mounted - two point hitch

In this arrangement the front of the implement is carried on the lower links of the tractor and the rear on a castor wheel as in Figure 6.2.

In the vertical, longitudinal plane the implement is free to pivot about the outer ends of the lower links and hence it behaves as the one point hitch above, ie, it is free to follow ground undulations. It is, however, rigid in the horizontal plane and is therefore frequently used for un-symmetrical implements having side forces, such as mouldboard or disc ploughs, or offset draught forces, such as forage mowers.

There is usually a significant static vertical component in the implement attachment force because part of the weight of the implement and of the downward soil forces are supported by the tractor. Thus weight transfer would be greater than in a corresponding traile implement; see Section 6.4.3.

#### (c) Fully mounted - three point hitch

Here the implement is attached to the tractor by means of the three-point linkage as shown in Figure 6.3. In this side view the lower two points are coincident; the upper point is midway between, but above the lower two.

This system totally constrains and allows complete control of the implement. It is not free to swing in space like the traile implement, nor in the vertical plane like the semi-mounted; it must operate in the position determined for it by the linkage. The exception to this statement is that the implement is usually free to rise, ie, it is not held down by the linkage. If it does rise, it will be due to the upward soil forces being greater than implement weight; it will, however, move in a way determined by the kinematics of the linkage.

In the vertical longitudinal plane (Figure 6.3) the linkage has the form of a mechanism known as a 'four link chain', the characteristics of which are treated in books on kinematics. We can identify the four links as shown in Figure 6.4:

- (i) the two lower links (which act as one in the vertical plane)
- (ii) the upper or top link
- (iii) the implement frame or pedestal
- (iv) the tractor chassis.

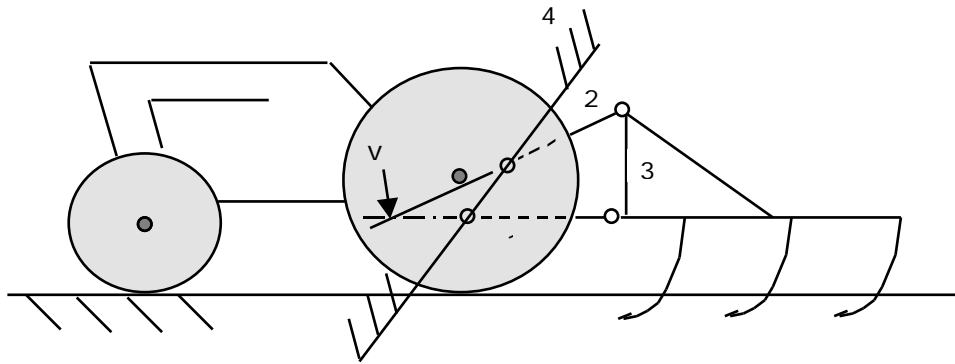


Figure 6.4: Three-point linkage as a four bar 'chain'.

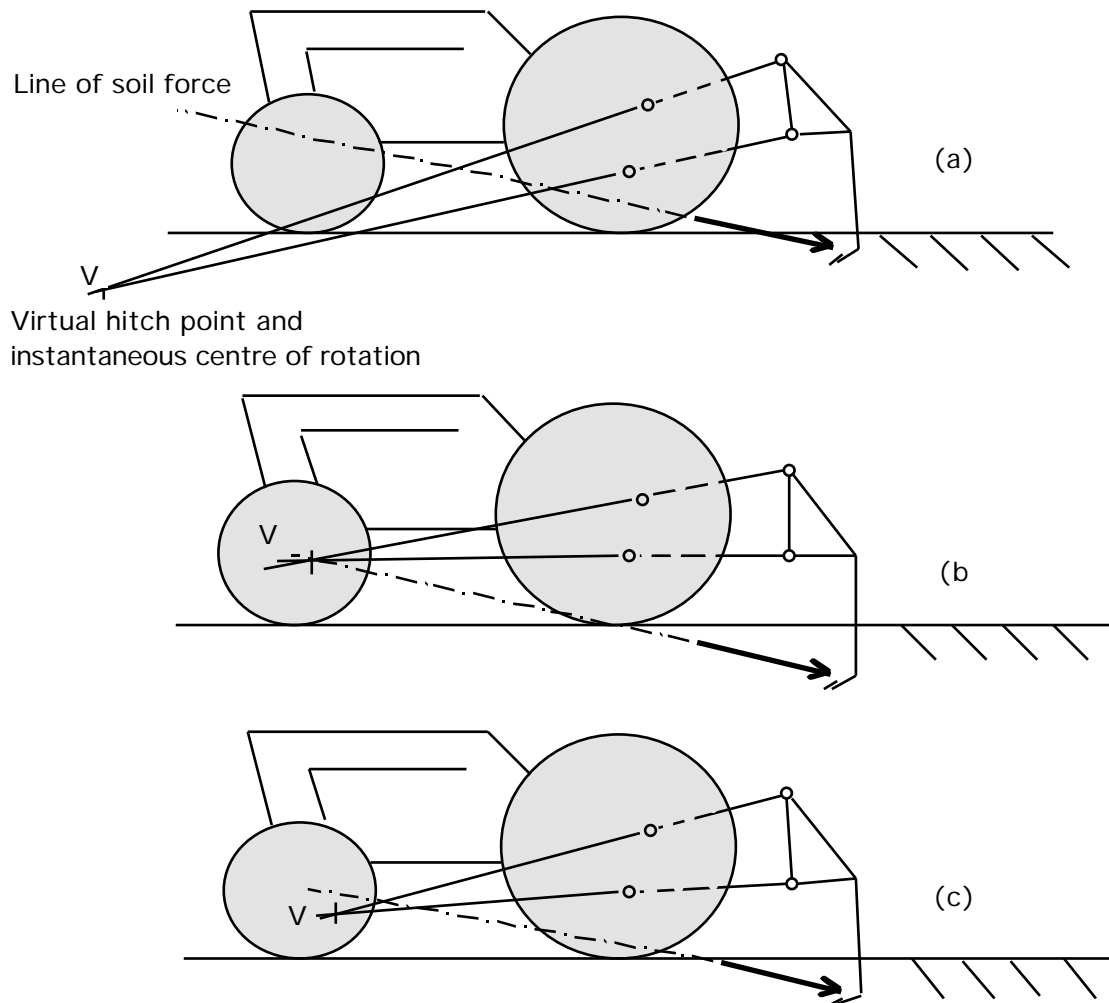


Figure 6.5: Three point linkage as an implement is lowered  
 (a) near commencement of penetration  
 (b) in a 'stable' free link condition  
 (c) restrained above the free link condition

The significant point is 'v' at the intersection of the upper and lower links. When discussing the motion of the implement it is termed the 'instantaneous centre of rotation'; at the instant shown, the implement moves as if it was rotating about that point. The point 'v' itself moves from instant to instant, hence the motion of the implement is quite complex.

When discussing the forces on the implement 'v' is termed the virtual or effective hitch point; at the instant shown, the implement behaves as if it were attached to the tractor at that point.

As an example, Figure 6.5(a) shows a plough on the three-point linkage as it enters the ground. It will be seen that the effective hitch point is below the ground and the line of draft passes above it. The soil force has a clockwise moment about that point, thus the plough is being pulled into the ground. As this occurs, the effective hitch point rises and eventually an equilibrium is reached where the downward force of soil on the plough is just balanced by the upward force of the tractor on the plough. The line of pull passes through the effective hitch point, now above the ground surface, as shown in Figure 6.5(b); this tends to add weight to the rear wheels of the tractor.

The above is termed the 'free link' condition but it is not suitable for normal operation because any variation in the direction of the soil force will cause the implement depth to change. Usually, the linkage is arranged so that the implement reaches the desired working depth before the effective hitch point rises up to the line of draft. The implement is thus kept from reaching the equilibrium condition; the soil forces tend to pull the plough in deeper, but the linkage stops this occurring. The weight of the plough and the downward acting soil forces are thus transferred to the rear wheels of the tractor. The line of draft passes above the effective hitch point, as shown in Figure 6.5(c); the former cannot be located from the latter as in the Figure 6.5(b). Further discussion is given in Dwyer (1974) and Inns (1985).

#### Problem 6.1

Take measurement of the three-point linkage system on a tractor and associated soil engaging implement. Plot on drawing paper the position of the instantaneous centre of rotation / virtual hitch point if the implement were raised and lowered to below the ground level. Alter the linkage or use another type of implement and repeat the above.

### 6.3 TRACTOR CHASSIS MECHANICS

The term 'mechanics' here refers to an analysis of the forces that act on the tractor chassis. The major force is that of gravity and is known as the weight. This is sometimes (loosely) given, and spoken of, in units of mass (kg); in engineering analysis (concerned with statics) all such 'weights' should be converted to force units (kN).

#### 6.3.1 Centre of gravity

The centre of gravity is the point at which the whole of the mass and the weight of the tractor may be considered to act. Its location depends on the disposition of the various masses that comprise the tractor. Any analysis of the tractor chassis requires the location of the centre of gravity to be known. It is usually specified in relation to the rear axle as shown by point G in Figure 6.6.

##### (a) Longitudinal location

The location of the centre of gravity in the longitudinal (x) direction may be found by measuring the weight on the front ( $W_f$ ) and rear ( $W_r$ ) wheels.

Application of the force equilibrium condition gives the tractor weight, W:

$$W = W_f + W_r$$

Application of the moment equilibrium condition gives the required longitudinal location,  $x_L$  as shown in Figure 6.6(a).

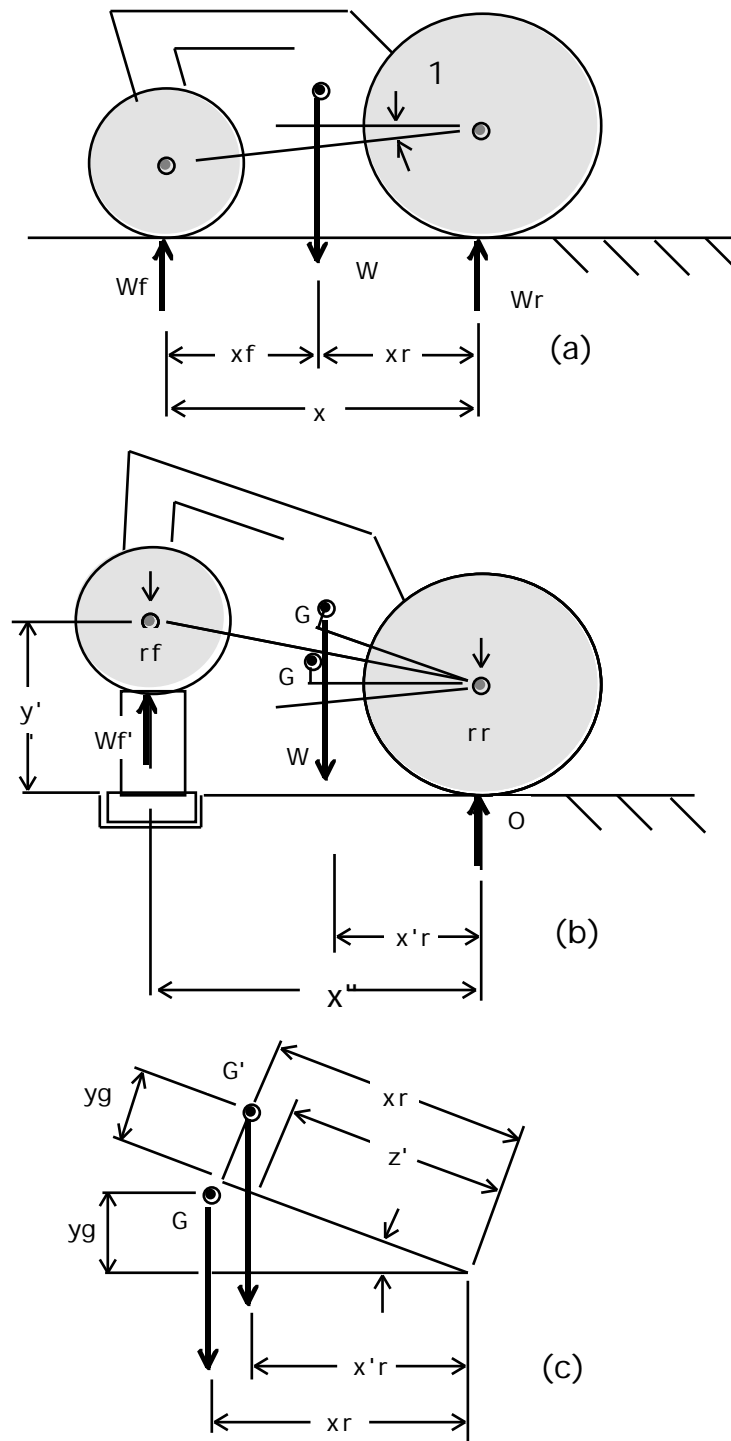


Figure 6.6: Location of centre of gravity of tractor  
 (a) horizontal location  
 (b) tractor raised to find vertical location  
 (c) geometry of position of centre of gravity  
 Adapted from Barger, et al (1952)

For the tractor take moments about O:

$$W \cdot x_r = W_f \cdot x$$

$$x_r = \frac{W_f}{W} x \quad (6.1)$$

The wheel base (x) between the front and rear axles is usually given in the manufacturer's specification or can be measured directly.

For most common rear wheel drive tractors  $x_r$  is approximately 30 % of x; this is also the % of the static tractor weight that is on the front wheels.

(b) Vertical location

The location of the centre of gravity in the vertical (y) direction is more difficult. The common method is to lift the front (or rear) of the tractor (as shown in Figure 6.6(b)) and measure the weight on the front wheels ( $W'_f$ ) in the raised condition. The following is similar to Barger et. al., (1952).

Application of the moment equilibrium condition gives the required vertical location,  $y_g$ .

For the tractor take moments about O:

$$x'_r = \frac{W'_f}{W} x'' \quad (6.2)$$

The geometry of the positions of the centre of gravity (Figure 6.1(c)) gives:

$$z = \frac{x'_r}{\cos}$$

$$y_g = \frac{x_r - z}{\tan}$$

Substituting for z gives

$$y_g = \frac{x_r - \frac{x'_r}{\cos}}{\tan} \quad (6.3)$$

where  $x'_r$  is as calculated from Equation 6.2 above.

$$\text{and} \quad = 1 + 2$$

$$= \text{atan} \frac{r_r - r_f}{x} + \text{atan} \frac{y' - r_r}{x''}$$

Inspection of Equation 6.3 shows that if the difference between  $x'_r$  and  $\frac{x'_r}{\cos}$  is to be accurately calculated, needs to be relatively large and / or accurately determined.

**Problem 6.2**

By a similar measurement and analysis to the above find the location in the vertical and longitudinal directions of the centre of gravity of a two wheeled tractor or trailer.



### 6.3.2 Issues in chassis mechanics

Two aspects of the mechanics of the tractor chassis, which are of importance to the performance of the tractor, can be identified:

#### (a) Weight transfer

For a tractor under dynamic (here meaning 'operating') conditions, the weight on the wheels will, in general, be different from the static values. These changes are termed 'weight transfer' although of course nothing is 'transferred'. The discussion here is limited to the changes in the vertical longitudinal plane, ie, from front to rear and vice versa because these have the greatest influence on tractor performance.

Weight transfer is a normal outcome of the action of the forces generated on the tractor chassis by the ground and by the implement. It occurs whenever and however the tractor is loaded, including the 'no' load case where there is some weight transfer due to the torque on the rear wheels required to propel the tractor against the rolling resistance of all the wheels..

It is also normally a desirable outcome because the tractor is designed to take advantage of it by having at least some of the driving wheels at the rear where, for normal forward operation, the increase in rear wheel weight is proportional to the drawbar pull. In reverse gear and in the 'over-run' condition, (the implement pushing the tractor) the forces toward the front of the tractor transfer weight from the rear wheels to the front wheels, a fact which affects the performance of the tractor in this type of work and when braking.

A more detailed discussion of the general subject of weight transfer is given in Gilfillan (1970), Liljedahl et al (1979) and other references given at the end of this Chapter.

#### (b) Instability

Instability occurs when the weight transfer is sufficient to cause the tractor to tip over rearwards. Impending instability (where the front wheels leave the ground and the tractor is on the point of becoming unstable) is considered here because it is a limiting case of the weight transfer and hence of tractor operation. It is an undesirable situation because it represents loss of steering control and may lead on directly to actual instability. Such a situation is partly avoided by inherent features of the design of the tractor-implement system and partly by its operation in a way that avoids reaching that condition. Usually the wheels slip before instability occurs.

An understanding of the actual process of tipping over in the vertical longitudinal plane which may follow requires a different, more complex dynamic analysis that includes, among other matters, the inertia of the tractor chassis and of the implement, also the inertia and stiffness of the transmission to the rear wheels. This and the analysis of instability in the lateral vertical plane (roll over) are not relevant to tractor performance as such; they are dealt with in Liljedahl et al (1979) and other references given at the end of this Chapter .

### 6.3.3 Analysis and assumptions

The following analysis of the tractor in the longitudinal, vertical plane is limited to the calculation of wheel weight during steady state operation in normal work (Section 6.4) and to the prediction of the conditions for impending instability (Section 6.5).

Although the tractor and implement are moving, the assumption of steady state operation implies that there are no inertia forces; the forces are doing external work but are not causing any acceleration. Hence the principles of statics and the conditions for static equilibrium of rigid bodies can be applied.

Three independent equations of equilibrium (chosen from the following) can be written:

- (i) the sum of the forces in any two perpendicular directions are zero. The two directions usually chosen are those parallel to and perpendicular to the ground surface.
- (ii) the sum of the moments about any two points in the vertical longitudinal plane are zero. The two points usually chosen are the wheel / ground contact points or the centres of the wheels.

In simple situations it may be sufficient to consider the whole tractor as a rigid body. Where the external forces are known the weights on the wheels can be calculated directly.

However it is sometimes convenient to consider the tractor as composed of two rigid bodies. One, the drive wheels, rotate about a centre located in the other - the chassis of the tractor. This occurs under the action of the torque acting on them which is internally produced by the engine. Any such analysis must apply appropriate constraints ie, that the forces and moments on each are equal and opposite.

In this analysis and the worked examples, the following simple assumptions are made:

- (i) forward motion is uniform; this assumes constant implement forces and no acceleration
- (ii) lines of forces on wheels are either tangential or radial or may be resolved as such; wheel sinkage and tyre distortion (but not normal tyre deflection) are neglected
- (iii) the tractor is symmetrical about the longitudinal vertical plane; all the forces and moments may be considered to act in this plane
- (iv) other forces, such as the change in position of the fuel and oil in the tractor on sloping ground, air resistance and other minor forces are neglected

The analyses of tractors where other more complex assumptions are made are given in the references at the end of this Chapter.

The tractor considered in the general analysis is as shown in Figure 6.7.

The implement force  $P$  acts through the point  $(x', y')$  at an angle to the ground surface. Note that it is not shown 'attached' to the chassis at the rear of the tractor because, in general, it may act on the tractor or attached implement at any point in the plane.

For a trailed hitch shown in Figure 6.1, this point would be the drawbar / implement attachment point. For the tractor in Problem 6.7,  $P$  is the weight of a tank and water (a vertical force) carried on the front. Care must therefore be taken to ensure that the direction and the moment of  $P$  is correctly included by appropriate choice of and the sign for  $x'$ .

The solution of the problems given in the following sections will be greatly facilitated by coding of Equations 6.4 and 6.5, etc, on a computer spread sheet.

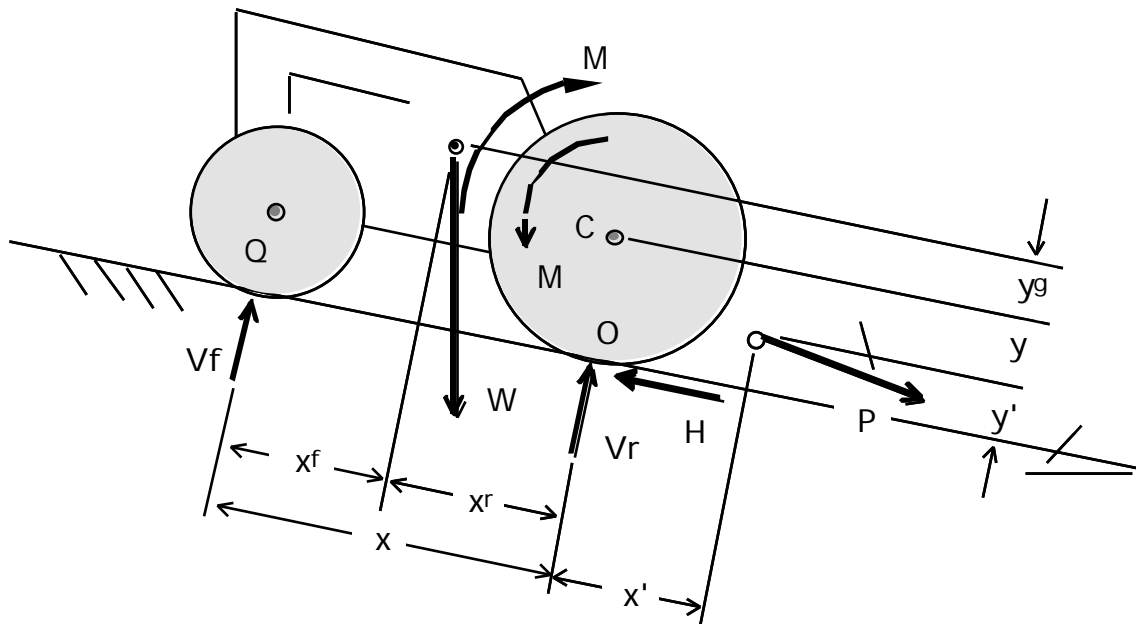


Figure 6.7: Tractor details for weight transfer analysis

## 6.4 WEIGHT TRANSFER

### 6.4.1 Four wheel tractor

#### (a) Analysis

Consider rear wheel drive tractor on a slope as shown in Figure 6.7.

For the tractor<sup>1</sup>, take moments about C:

$$V_f x + W \sin y g + P \sin x' + M = W \cos x_r + P \cos y$$

$$V_f = W \cos \frac{x_r}{x} + P \cos \frac{y}{x} - \frac{M}{x} - W \sin \frac{y_g}{x} - P \sin \frac{x'}{x}$$

For the wheels, take moments about C:

$$M = H \cdot r$$

Resolve parallel to the slope:

$$H = W \sin + P \cos$$

Substitute for M and H above:

$$V_f = W \cos \frac{x_r}{x} + P \cos \frac{y}{x} - W \sin \frac{r}{x} - P \cos \frac{r}{x} - W \sin \frac{y_g}{x} - P \sin \frac{x'}{x}$$

Combining:

$$V_f = W \cos \frac{x_r}{x} - W \sin \frac{r+y_g}{x} - P \cos \frac{r-y}{x} - P \sin \frac{x'}{x}$$

$$V_f = W_f - W \sin \frac{r+y_g}{x} - P \cos \frac{y'}{x} - P \sin \frac{x'}{x} \quad (6.4)$$

#### **Problem 6.3**

Show that the weight on the rear wheels ( $V_r$ ) perpendicular to the slope is given by:

$$V_r = W_r + W \sin \frac{r+y_g}{x} + P \cos \frac{y'}{x} + P \sin \frac{x+x'}{x} \quad (6.5)$$

<sup>1</sup> In the following, the total weight of the tractor ( $W$ ) and the distance to its centre of gravity ( $x_r$ ) have been used; this is statically equivalent to using the weight of the body (tractor less rear wheels) and the distance to its centre of gravity.

(b) Explanation of terms

The terms in Equations 6.4 and 6.5 can be identified as follows:

- (i)  $W_f, W_r$  the static weight on the wheels when the tractor is on the slope
- (ii)  $W \sin \frac{r+y}{x}$  the moment effect of the weight component down the slope, decreasing the front wheel weight and increasing the rear.
- (iii)  $P \cos \frac{y'}{x}$  the moment effect of the implement force component down the slope, decreasing the front wheel weight and increasing the rear.
- (iv)  $P \sin \frac{x'}{x}$  the moment effect of the implement force component perpendicular to the slope, decreasing the front wheel weight.
- (v)  $P \sin \frac{x+x'}{x}$  the direct ( $P \sin$ ) and the moment effect ( $P \sin \frac{x'}{x}$ ) of the implement force component perpendicular to the slope, increasing the rear wheel weight.

Referring to the Equations 6.4 and 6.5, note that the moment effect of the component of the drawbar pull down the slope,  $P \cos$ , has two effects:

- (i)  $P \cos \frac{y}{x}$ : increases  $V_f$  and decreases  $V_r$  with moment arm  $y$
- (ii)  $P \cos \frac{r}{x}$  decreases  $V_f$  and increases  $V_r$  with moment arm  $r$

The net effect of  $P \cos$  is therefore the difference between these two, ie,  $P \cos \frac{r-y}{x} = P \cos \frac{y'}{x}$ .

This fact gives rise to the idea that if the drawbar pull acts below the rear axle, its moment,  $P \cos \cdot y$ , increases  $V_f$  and holds the front of the tractor down. While this is true, it omits the more important, unrecognised aspect that a usually larger moment,  $P \cos \cdot r$ , tends to decrease the weight on the front wheels.

**Problem 6.4**

Check Equations 6.4 and 6.5 by taking moments about the ground contact points O and Q, respectively.

(c) Special cases

The following special cases are of interest:

- (i) If  $y'$  increases, ie, the point of action (eg, the drawbar) is raised,  $y$  decreases and the weight transfer,  $P \cos \frac{r-y}{x}$  increases; the tractor may reach the condition of impending instability when  $V_f = 0$  (Refer Section 6.5)
- (ii) If  $y' = 0$ , the point of action (the drawbar) is at ground level,  $y = r$ ; there is no weight transfer due to  $P$ .
- (iii) If  $y'$  is negative, the point of action is below ground level (eg, as is possible with a three point linkage or with the drawbar in a trench),  $y$  is greater than  $r$ , the term  $P \cos \frac{y'}{x}$  becomes positive in Equation 6.4 and negative in Equation 6.5, ie, weight is transferred from the rear to the front wheels. (Refer Section 6.4.3.
- (iv) If  $\theta = 0$ , ie, the implement force is parallel to the ground

$$V_f = W \cos \frac{x_r}{x} - W \sin \frac{r+y_g}{x} - P \frac{y'}{x} \quad V_r = W \cos \frac{x_f}{x} + W \sin \frac{r+y_g}{x} + P \frac{y'}{x}$$

- (v) If also,  $\theta = 0$ , ie, the ground is horizontal

$$V_f = W \frac{x_r}{x} - \frac{Py'}{x} = W_f - \frac{Py'}{x} \quad V_r = W \frac{x_f}{x} + \frac{Py'}{x} = W_r + \frac{Py'}{x}$$

- (vi) If also,  $P = 0$ , ie, there is no implement force

$$V_f = W \frac{x_r}{x} = W_f \quad V_r = W \frac{x_f}{x} = W_r$$

**Problem 6.5**

Repeat the analysis in Section 6.4.1 for the tractor travelling down the slope where the implement force acts forwards and downwards (as when towing an unbalanced trailer); show that the wheel weights are:

$$V_f = W_f + W \sin \frac{r+y_g}{x} + P \cos \frac{y'}{x} - P \sin \frac{x'}{x} \quad (6.6)$$

$$V_r = W_r - W \sin \frac{r+y_g}{x} - P \cos \frac{y'}{x} + P \sin \frac{x'+x}{x} \quad (6.7)$$

**Problem 6.6**

Consider the Farmland tractor with a spray tank mounted on the three-point linkage at the rear.

The following data apply:

Weight of spray tank when empty	= 60 kg
Centre of gravity of the tank and water	= 1.5 m from the rear axle
	= 1.0 m from the ground

- (i) If there is 210 kg of water in the tank, what is the weight on the front wheels for the unit moving on horizontal ground?
- (ii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving up a  $10^\circ$  slope and the weight on the front wheels is to not be less than 4kN?
- (iii) What will be the maximum weight on the front wheels and the tractive coefficient as the tractor empties the spray tank while travelling down a  $10^\circ$  slope ?

**Solution Part (ii)**

From Equation 6.4:

$$V_f = W_f - W \sin \frac{r+y_g}{x} - P \cos \frac{y'}{x} - P \sin \frac{x'}{x}$$

$$P = \frac{W \cos \frac{x_r}{x} - W \sin \frac{(r+y_g)}{x} - V_f x}{\cos y' + \sin x'}$$

$$= \frac{27.9 (.532 - .133) - 7.52}{.174 + 1.48} = 2.18 \text{ kN} = 224 \text{ kg}$$

Weight of water =  $224 - 60 = 164 \text{ kg}$

From Equation 6.5

$$V_r = W_r + W \sin \frac{r+y_g}{x} + P \cos \frac{y'}{x} + P \sin \frac{x+x'}{x}$$

$$= 27.9(.985 \frac{1.34}{1.88} + .174 \frac{.765}{1.88}) + 2.18 (.174 \frac{1}{1.88} + .985 \frac{3.38}{1.88})$$

$$= 19.6 + 1.97 + .20 + 3.89$$

$$= 25.6 \text{ kN}$$

$$\mu = \frac{W \sin \frac{r+y_g}{x} + P \cos \frac{y'}{x}}{V_r} = \frac{27.9 \times .174 + 2.18 \times .174}{25.6} = 0.20$$

**Answers:** (i) 5.92 kN; (ii) 164 kg, 0.20; (iii) 9.48kN, -0.27

**Problem 6.7**

Repeat Problem 6.6 with the spray tank mounted on the front of the tractor with its centre of gravity 1.5 m from the front axle.

- (i) If there is 210 kg of water, what is the weight on the front wheels for the unit moving on horizontal ground?
- (ii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving up a  $10^\circ$  slope and the front wheel weight is to not exceed 10 kN?
- (iii) What weight of water can be carried and what will be the tractive coefficient (based on the total tractive force) if the unit is moving down a  $10^\circ$  slope and the front wheel weight is to not exceed 14 kN?

**Answers:** (i) 12.8 kN; (ii) 187 kg, 0.26; (b) 165 kg; -0.33

**Problem 6.8**

Consider the Farmland tractor operating up a slope  $= 15^\circ$  with a drawbar pull angle  $= 10^\circ$ .

Use Equation 6.5 to calculate the :

- (i) maximum drawbar pull if the tractive coefficient (based on the total tractive force) = 0.8
- (ii) rear wheel weight
- (iii) percentage contributions of the terms in Equation 6.5 to the tractive force.

Note: An iterative method is required to solve this problem because the rear wheel weight depends on the drawbar pull (due to weight transfer) and the drawbar pull (as determined by the tractive coefficient) depends on the rear wheel weight. Assume an initial value for  $P$  and calculate  $V_r$ ,  $H$  and then  $P$ ; if the initial value of  $P$  is carefully chosen, the answer will be obtained with sufficient accuracy with two iterations.

**Answers:** (i) 17.1 kN; (ii) 30.1 kN; (iii) 64%, 10%, 13%, 13%

**6.4.2 Weight transfer with rolling resistance**

The above analysis neglects any effect of rolling resistance. We may, however, include this by introducing a force acting along the slope (opposite the direction of motion) as a further force to be overcome by the tractor.

As discussed in Section 4.3.3 the rolling resistance may be expressed in terms of a coefficient ( ) as

$$\text{Rolling resistance} = \quad . \text{ Weight on wheel}$$

Here the weight will be the wheel weights perpendicular to the slope, ie,  $V_f$  and  $V_r$  as given by Equations 6.4 and 6.5 above. The rolling resistance for the tractor may be estimated by combining the effect on the front and rear wheels by considering a coefficient for the tractor as a whole.

$$\begin{aligned} R &= (V_f + V_r) \\ &= (W \cos \quad + P \sin \quad) \end{aligned}$$

The total tractive force

$$H = W \sin \quad + P \cos \quad + (W \cos \quad + P \sin \quad)$$



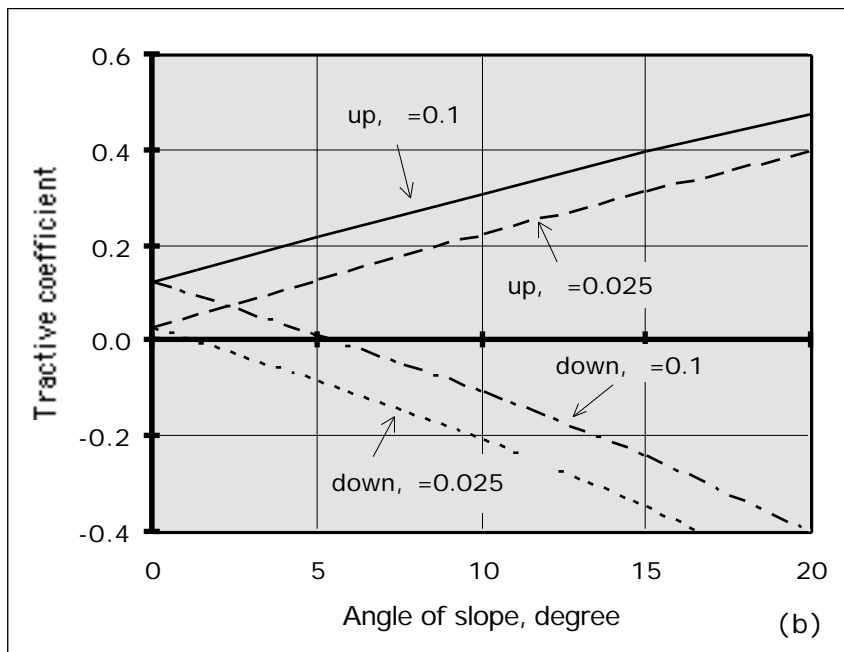
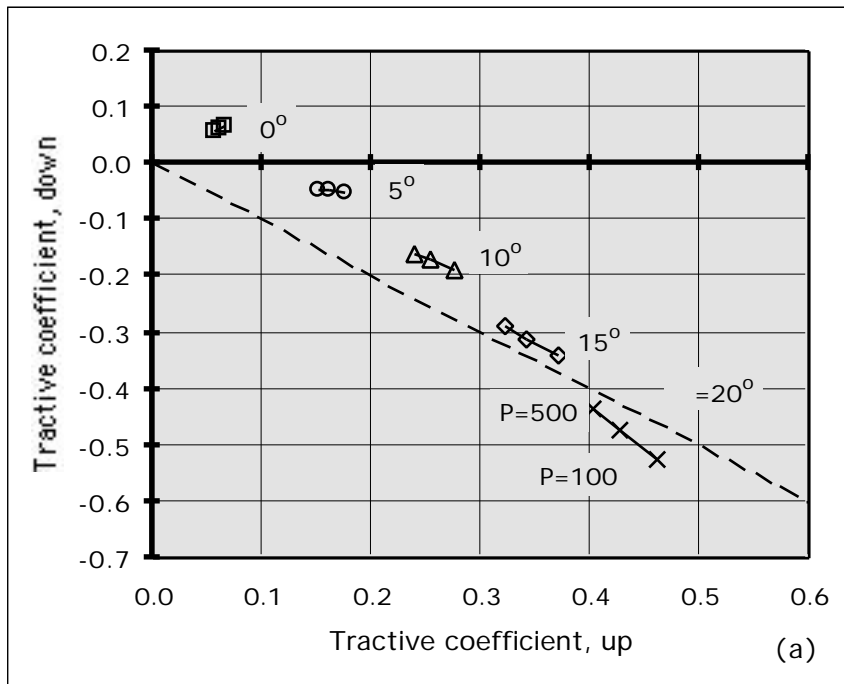


Figure 6.8: Tractive coefficients required for the Farmland tractor working up and down the slope :  
 (a) carrying a weight of 100, 300 and 500kg with rolling resistance coefficient of 0.05  
 (b) Carrying a weight of 300kg with rolling resistance coefficient of 0.025 (bitumen road) and 0.1 (ploughed soil).

We can specify the tractive force required (for a rear wheel drive tractor) in terms of the gross tractive coefficient.

$$\begin{aligned} \mu &= \frac{\text{Tractive force}}{\text{Rear wheel weight}} \\ &= \frac{W \sin \theta + P \cos \theta + (W \cos \theta + P \sin \theta)}{W_r + W \sin \theta \frac{r+y_g}{x} + P \cos \theta \frac{y'}{x} + P \sin \theta \frac{x+x'}{x}} \end{aligned} \quad (6.8)$$

### Problem 6.9

The Farmland tractor carries a fertilizer distributor mounted on the rear three-point linkage. The following data apply:

Centre of gravity of distributor and fertilizer, m: 1.5 behind tractor rear axle  
1.0 above ground

Total weight of the distributor and fertilizer, kg: 100 (empty),  
300  
500 (full)

Rolling resistance coefficient 0.025 (bitumen road),  
0.050 (firm surface)  
0.1 (ploughed soil)

Angle of slope (up and down), ° 0, 5, 10, 15, and 20

Calculate the traction coefficient required to drive the tractor and distributor under various conditions. Hence identify conditions where it may be possible and safe to drive up a slope but unsafe to drive down it.

### Solution

Results for some conditions which are given in Figure 6.8(a) for  $r=0.05$  (firm conditions) show that the tractive coefficient required:

- (i) increases with the angle of slope
- (ii) decreases with weight carried, particularly for larger angles

Figure 6.8(b) shows that the tractive coefficient depends on the angle of slope and the rolling resistance. In the example given for load = 300 kg and  $r = 0.025$  (bitumen road),  $\mu$  (down) >  $\mu$  (up) for slope > 12°.

### Problem 6.10

Repeat Problem 6.9 with the distributor mounted on the front of the tractor.

Assume that the centre of gravity of distributor and fertilizer is 1.5 m in front of the front axle and 1.0 m above the front wheel ground contact point.

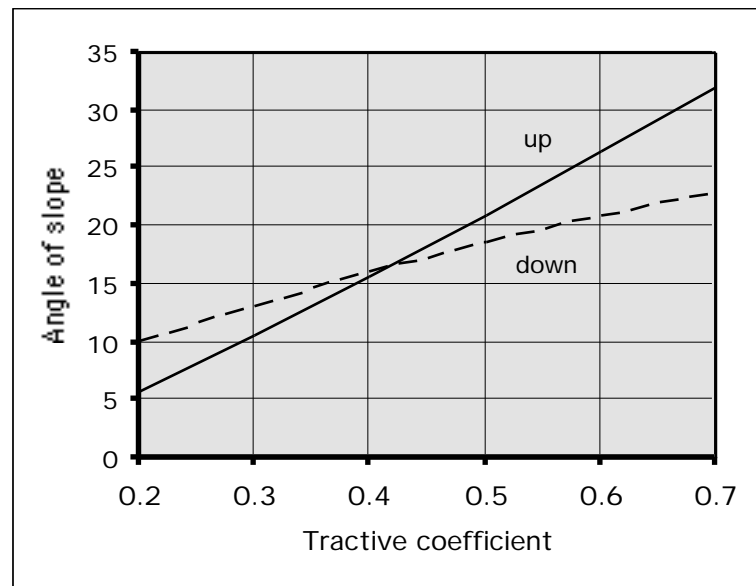


Figure 6.9: Slopes that can be negotiated for various traction coefficients, Problem 6.11

**Problem 6.11**

The Farmland tractor operates with zero drawbar pull on a slope  $\alpha$ . The maximum gross tractive coefficient is  $\mu$  and the coefficient of rolling resistance for the tractor as a whole is  $r$ .

(i) What is the maximum slope that the tractor can travel up without exceeding the maximum tractive force.

Resolving along the slope, Figure 6.7:

$$H = W \sin \alpha + W \cos \alpha r$$

At maximum gross tractive coefficient :

$$H = V_r \mu$$

The dynamic weight  $V_r$  on the rear wheels in operation is given by moments about Q:

$$V_r \cdot x = W \cos \alpha x_f + W \sin \alpha (r + y_g)$$

$$V_r = \frac{W \cos \alpha x_f + W \sin \alpha (r + y_g)}{x}$$

Substitute for  $H$  and  $V_r$  above:

$$W \sin \alpha + W \cos \alpha r = \left[ \frac{W \cos \alpha x_f + W \sin \alpha (r + y_g)}{x} \right] \mu$$

$$W \sin \alpha \left[ 1 - \frac{(r + y_g)}{x} \mu \right] = W \cos \alpha \left[ \frac{x_f}{x} \mu - r \right]$$

$$\tan \alpha_u = \frac{\mu x_f - x}{x - \mu(r + y_g)} \quad (6.9)$$

(ii) Show that the maximum slope that the tractor can travel down without exceeding the maximum tractive force is:

$$\tan \alpha_d = \frac{\mu x_f + x}{x + \mu(r + y_g)} \quad (6.10)$$

(iii) Plot  $\tan \alpha_u$  and  $\tan \alpha_d$  for values of  $\mu$  between 0.2 and 0.7 and  $r = 0.05$  and discuss the meaning of these results.

Answer (iii) See Figure 6.9

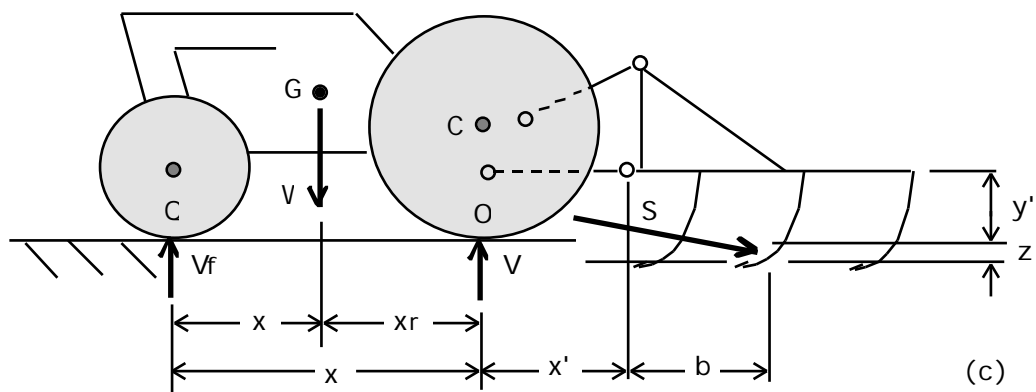
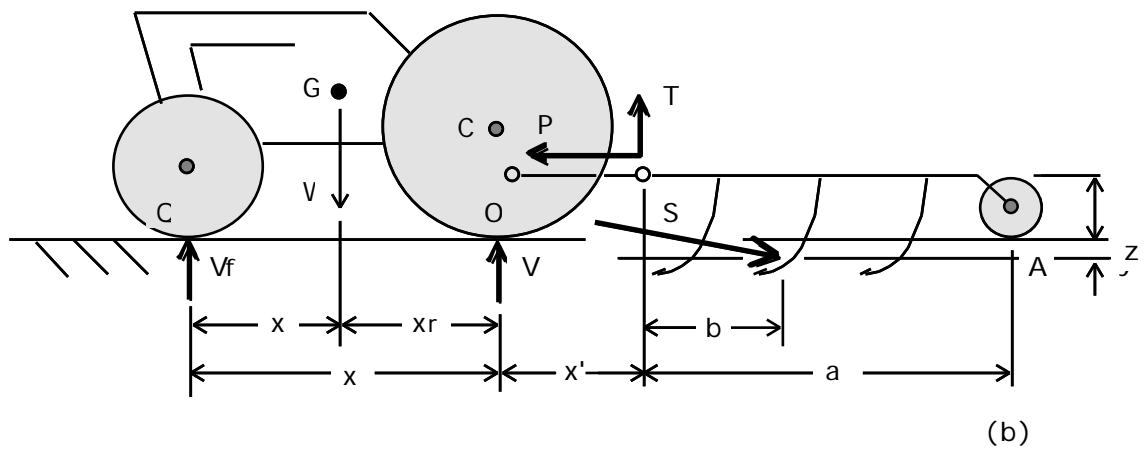
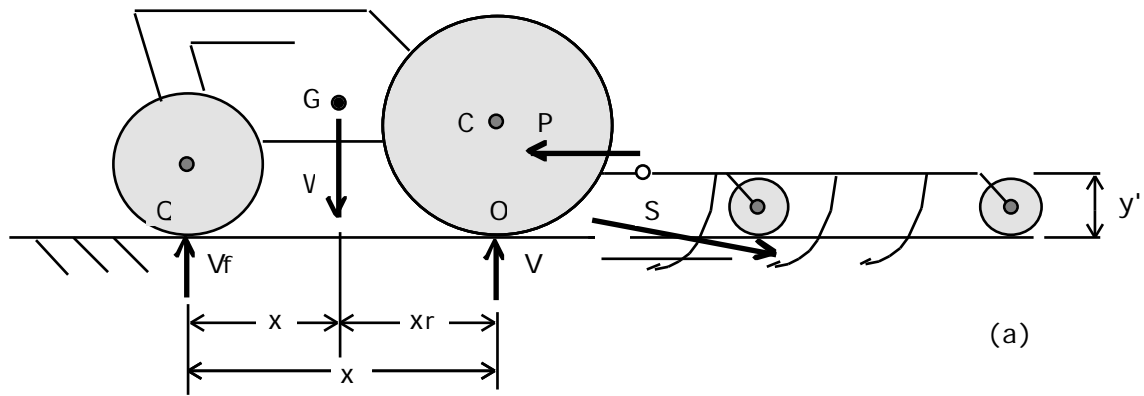


Figure 6.10: Weight transfer with various hitching systems;  
(a) trailed; (b) semi-mounted; (c) fully mounted

### 6.4.3 Weight transfer with hitching systems

#### (a) Analysis

Considering the three common hitching systems described in Section 6.2.2 above, we are now in a position to evaluate them with respect to weight transfer, ie, the increase in the weight on the rear wheels as a result of the implement forces. This analysis does not take into account the weight of the implement, which is more significant for the mounted and semi-mounted systems than for the trailed. However, it provides a valid comparison of the relative advantages of weight transfer of the three systems on the basis of the soil forces and of the conditions under which these advantages will be achieved.

Consider an identical cultivator, as shown in Figure 6.10 hitched, in the following ways:

- (i) trailed on its own wheels
- (ii) semi-mounted on the lower links of the tractor and a rear wheel
- (ii) fully mounted on the three-point linkage.

In order to compare them it is necessary to determine the dynamic weight on the front and rear wheels of the tractor for each system; the same soil force  $S$ , acting at an angle to the ground surface as shown, is assumed for each.

#### (i) *Trailed*

Resolving horizontally:

$$P = S \cos$$

Moments about Q for the tractor:

$$V_r x = W x_f + P y'$$

$$V_r = W_r + \frac{S \cos y'}{x} \quad (6.11)$$

And

$$V_f = W_f - \frac{S \cos y'}{x} \quad (6.12)$$

Weight transfer will occur if  $V_r > W_r$  ie, if  $y'$  is positive, ie, if the drawbar is above ground level; it will be increased by increasing the drawbar height,  $y'$ .

For a consideration of the implications of this, see the more general analysis of impending instability given in Section 6.5.

#### (ii) *Semi-mounted*

Resolving horizontally:

$$P = S \cos$$

The dynamic weight  $T$  on the tractor drawbar is given by moments about A for the cultivator:

$$T a = S \sin (a-b) + P y' + S \cos z$$

where  $b$  gives the horizontal location of the soil force.

Substituting for  $P$

$$T = S \sin \frac{a-b}{a} + S \cos \frac{z+y'}{a}$$

The dynamic weight on the rear wheels is given by moments about Q:

$$V_r x = W x_f + P y' + T (x+x')$$

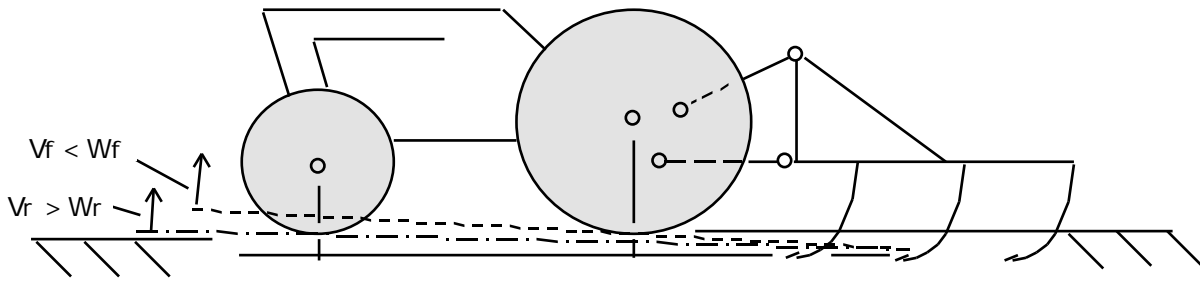


Figure 6.11 Conditions for weight transfer with fully mounted implement.

HITCHING SYSTEM	CONDITION FOR WEIGHT TRANSFER	EXPLANATION
TRAILED	$V_r > W_r$ unless $y'$ negative	Drawbar above ground level
SEMI MOUNTED	$V_r > W_r$ always	- -
MOUNTED	$V_r > W_r$ if $\tan > \frac{z}{x+x'+b}$ $V_f < W_f$ if $\tan > \frac{z}{x'+b}$	Line of soil force passes above:- front wheel/ ground contact point rear wheel / ground contact point

Table 6.1: Summary of conditions for weight transfer with various hitching systems

Substituting for T and P:

$$\begin{aligned}
 V_r x &= W x_f + S \cos y' + S \sin \frac{(a-b)(x+x')}{a} + S \cos \frac{(z+y')(x+x')}{a} \\
 V_r &= W_r + S \sin \frac{(a-b)(x+x')}{ax} + S \cos \left[ \frac{y'}{x} + \frac{(z+y')(x+x')}{ax} \right] \\
 &= W_r + S \sin \frac{(a-b)(x+x')}{ax} + S \cos \frac{ay' + (z+y')(x+x')}{ax} \quad (6.13)
 \end{aligned}$$

### Problem 6.12

Show that the weight on the front wheels of the tractor with semi-mounted implement is given by:

$$V_f = W_f - S \sin \frac{(a-b)x'}{ax} - S \cos \frac{ay' + (z+y')x'}{ax} \quad (6.14)$$

Weight transfer will occur if  $V_r > W_r$  which will always occur unless one of the following terms is negative and greater in magnitude than the other.

The first term will be negative if  $b > a$ , ie, the soil force is behind the wheel. The second will be negative if  $y'$  is negative (below ground level) and greater than  $z$  or  $z$  is negative (above ground level) and greater than  $y'$ .

All of these conditions are unlikely to occur for a semi-mounted implement, hence weight transfer will always occur.

### (iii) Mounted

The dynamic weight  $V_r$  on the rear wheels is given by moments about Q for the tractor / implement system as a whole:

$$\begin{aligned}
 V_r x + S \cos z &= W x_f + S \sin (x+x'+b) \\
 V_r &= W_r + S \sin \frac{x+x'+b}{x} - S \cos \frac{z}{x} \quad (6.15)
 \end{aligned}$$

The dynamic weight  $V_f$  on the front wheels is given by moments about O for the tractor / implement system as a whole:

$$\begin{aligned}
 W x_r + S \cos z &= V_f x + S \sin (x'+b) \\
 V_f &= W_f - S \sin \frac{x'+b}{x} + S \cos \frac{z}{x} \quad (6.16)
 \end{aligned}$$

Increasing the length of mounted implements (hence increasing  $b$ ) will increase the weight transfer to the rear wheels due to the direct effect ( $S \sin$ ) and the moment effect ( $S \sin \frac{x'+b}{x}$ ) from the front wheels. The limit will be the length and weight that will still allow the tractor to lift the implement without itself tipping up; weights may be added to the front of the tractor to avoid this.



Weight transfer will occur if:

$$\begin{aligned}
 V_r &> W_r \\
 S \sin \frac{x+x'+b}{x} &> S \cos \frac{z}{x} \\
 \tan &> \frac{z}{x+x'+b}
 \end{aligned} \tag{6.17}$$

This implies that weight transfer to the rear wheels will occur if the soil force passes above the front wheel / ground contact point, Figure 6.11.

The above includes the contribution of the vertical component of the soil force ( $S \sin$ ) to the rear wheel weight.

Another measure associated with weight transfer from the front wheels in the mounted system is the condition that

$$\begin{aligned}
 V_f &< W_f \\
 S \sin \frac{x'+b}{x} &> S \cos \frac{z}{x} \\
 \tan &> \frac{z}{x'+b}
 \end{aligned} \tag{6.18}$$

This implies that weight transfer from the front wheels to the rear will occur if the soil force passes above the rear wheel / ground contact point. Further, weight transfer will increase as  $b$  increases, ie, the implement gets longer.

#### (iv) Summary

A summary of the results of this analysis is given in Table 6.1.

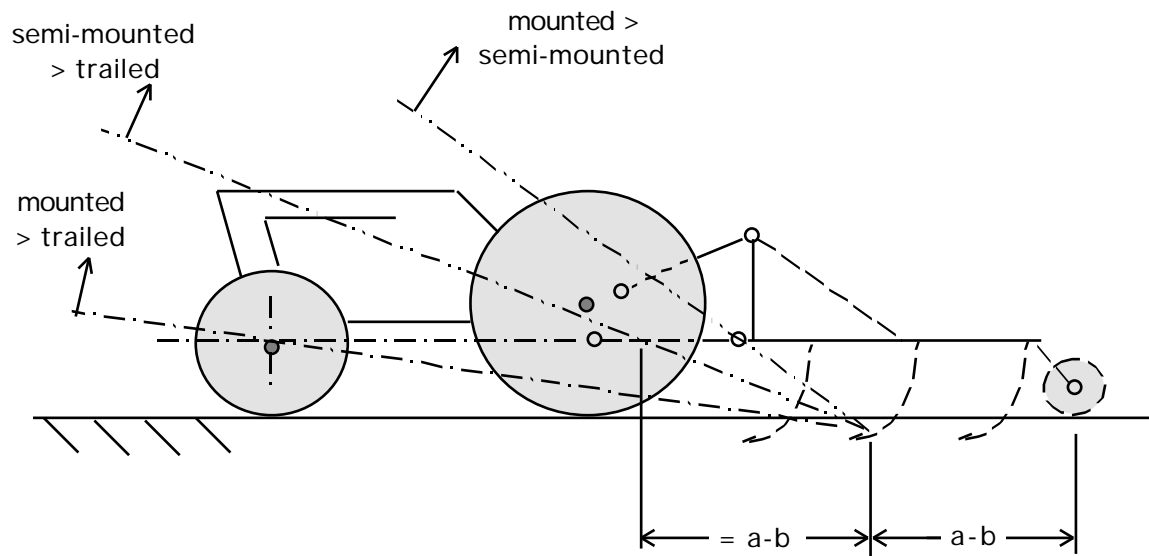


Figure 6.12: Comparison of hitching systems on the basis of weight transfer

TOP GREATER THAN SIDE if line of soil force passes above:	MOUNTED	
SEMI MOUNTED	Axis of lower hitch points: $\tan > \frac{z+y'}{b}$	SEMI MOUNTED
TRAILED	Intersection, drawbar line & vertical line through front axle: $\tan > \frac{z+y'}{x+x'+b}$	Intersection, drawbar line & vertical line as far in front of soil force as wheel is behind it: $\tan > \frac{z+y'}{a-b}$

Table 6.2 Summary comparison of weight transfer effects for different hitching systems

(b) Comparison of hitching systems

We seek to determine the conditions under which the weight transfer for each system in Section 6.4.3 (a) is greater than the one above it.

(i) *Condition for  $V_r(\text{mounted})$  greater than  $V_r(\text{semi-mounted})$  :*

$$\begin{aligned}
 S \sin \frac{x+x'+b}{x} - S \cos \frac{z}{x} &> S \sin \frac{(a-b)(x+x')}{ax} + S \cos \left[ \frac{ay' + (z+y')(x+x')}{ax} \right] \\
 \sin \left[ \frac{x+x'+b}{x} - \frac{(a-b)(x+x')}{ax} \right] &> \cos \left[ \frac{z}{x} + \frac{ay' + (z+y')(x+x')}{ax} \right] \\
 \sin [b(a+x+x')] &> \cos [(z+y')(a+x+x')] \\
 \tan &> \frac{(z+y')}{b}
 \end{aligned} \tag{6.19}$$

For the weight transfer of the mounted implement to be greater than that for the semi-mounted, the soil force must pass above the lower hitch points; see Figure 6.12.

(ii) *Condition for  $V_r(\text{mounted})$  greater than  $V_r(\text{trailed})$  :*

$$\begin{aligned}
 S \sin \frac{x+x'+b}{x} - S \cos \frac{z}{x} &> S \cos \frac{y'}{x} \\
 \sin \frac{x+x'+b}{x} &> \cos \frac{z+y'}{x} \\
 \tan &> \frac{z+y'}{x+x'+b}
 \end{aligned} \tag{6.20}$$

For the weight transfer for the mounted implement to be greater than that for the trailed, the soil force must pass above the intersection of the drawbar line and a vertical line through the front axle; see Figure 6.12.

(iii) *Condition for  $V_r(\text{semi-mounted})$  greater than  $V_r(\text{trailed})$  :*

$$\begin{aligned}
 S \sin \frac{(a-b)(x+x')}{ax} + S \cos \left[ \frac{ay' + (z+y')(x+x')}{ax} \right] &> S \cos \frac{y'}{x} \\
 \sin \frac{(a-b)(x+x')}{ax} &> \cos \left[ \frac{y'}{x} - \frac{ay' + (z+y')(x+x')}{ax} \right] \\
 \tan &> \frac{z+y'}{a-b}
 \end{aligned} \tag{6.21}$$

For the weight transfer for the semi-mounted cultivator to be greater than that for the trailed, the soil force must pass above the intersection of the drawbar line and vertical line through a point as far forward of the soil force as the wheel of the semi-mounted cultivator is behind it; see Figure 6.12.

(iv) *Summary*

A summary of the results of this analysis is given in Table 6.2

The above conditions are likely to be met with implements which have;

- (i) a soil force with significant vertical component, such as mouldboard ploughs, compared to those with a more horizontal force, such as cultivators.
- (ii) long implements for which  $b$  is large.

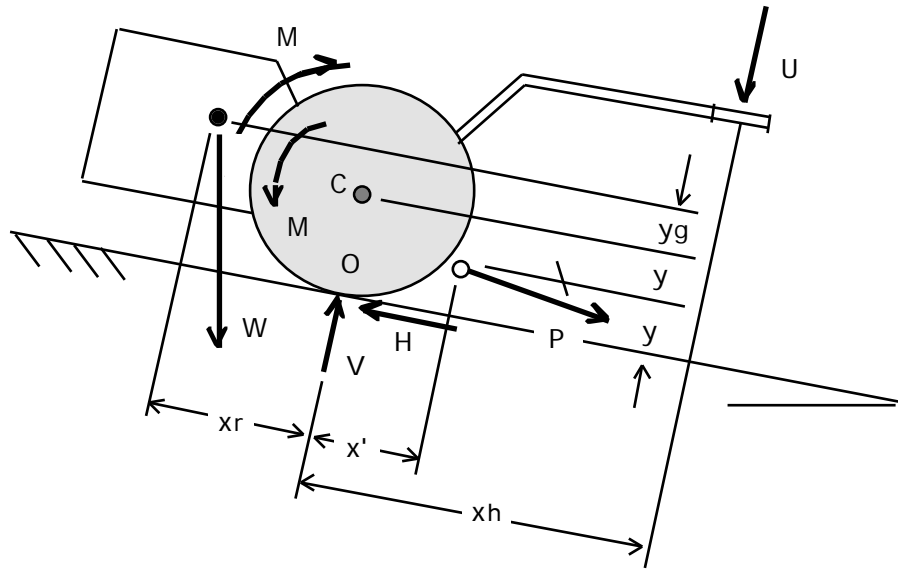


Figure 6.13 Two wheeled tractor dimensions relevant to weight transfer analysis

### 6.4.4 Other examples

#### (a) Two wheel (walking) tractor

The wheels of a two wheel ( or so-called 'walking') tractor are usually driven through a V belt and / or chain drive as shown in Figure 1.3. The mechanics of its chassis are the same, in principle, as the conventional four-wheel tractor but here the tractor chassis requires 'support' when pulling a drawbar load. This will usually be provided by one or more of the following:

- (i) a tool, implement or trailer at the rear
- (ii) a wheel at the rear
- (iii) a counter balance weight at the front
- (iv) the operator through the handles

Consider the two-wheel tractor as shown in Figure 6.13 on a slope with an angled pull through the drawbar. Normally the location of the centre of gravity would be such that with no drawbar pull the tractor would tip forwards and a counteracting force  $U$  acting down on the handles would be required. When a drawbar pull acts the net moment on the chassis will be clockwise as in Figure 6.13 and so the tractor tends to balance itself.

#### (i) *With zero drawbar pull:*

For the tractor, take moments about O:

$$U_o x_h + W \sin (r + y_g) = W \cos x_r$$

$$U_o = W \cos \frac{x_r}{x_h} - W \sin \frac{r + y_g}{x_h} \quad (6.22)$$

This force must act downward as shown if the centre of gravity of the tractor, counter weight and implement are forward of the axle.

#### (ii) *With drawbar pull:*

For the tractor resolve parallel to the slope:

$$H = P \cos + W \sin$$

Take moments about C for the wheels:

$$M = H \cdot r$$

Moments about C for the tractor:

$$M + W \sin y_g + P \sin x' + U x_h = W \cos x_r + P \cos (r-y')$$

Substitute for H and M from above:

$$P \cos r + W \sin r + W \sin y_g + P \sin x' + U x_h = W \cos x_r + P \cos (r-y')$$

$$U = W \cos \frac{x_r}{x_h} - W \sin \frac{r + y_g}{x_h} - P \cos \frac{y'}{x_h} - P \sin \frac{x'}{x_h} \quad (6.23)$$

$$= U_o - P \cos \frac{y'}{x_h} - P \sin \frac{x'}{x_h} \quad (6.24)$$

**Problem 6.13**

For the two-wheel tractor on slope and with angled drawbar pull, show that the normal wheel weight is:

$$V = W \cos \frac{x_h + x_r}{x_h} - W \sin \frac{r+y_g}{x_h} - P \cos \frac{y'}{x_h} + P \sin \frac{x_h - x'}{x_h} \quad (6.25)$$

For the convenient operation of such a tractor it would be desirable to arrange that the force  $U = 0$  under operating conditions. Examination of Equation 6.23 (for simplicity with  $\theta = 0$ ) shows that this will depend on balancing the moment of the weight and of the drawbar pull.

$$W \cos \frac{x_h + x_r}{x_h} = P \cos \frac{y'}{x_h} + P \sin \frac{x_h - x'}{x_h}$$

To achieve this it is common to attach a large weight at the front of the tractor, the position of which is adjustable with respect to the axle (equivalent to changing  $x_r$ ) to achieve the desired balance.

$$x_r = \frac{P \cos \frac{y'}{x_h} + P \sin \frac{x_h - x'}{x_h}}{W \cos \frac{x_h + x_r}{x_h}}$$

**Problem 6.14**

Show that for the walking tractor with  $\theta = 0$  and  $\phi = 0$ , the condition for  $U=0$  at maximum drawbar pull is that  $x_r = x' y'$ .

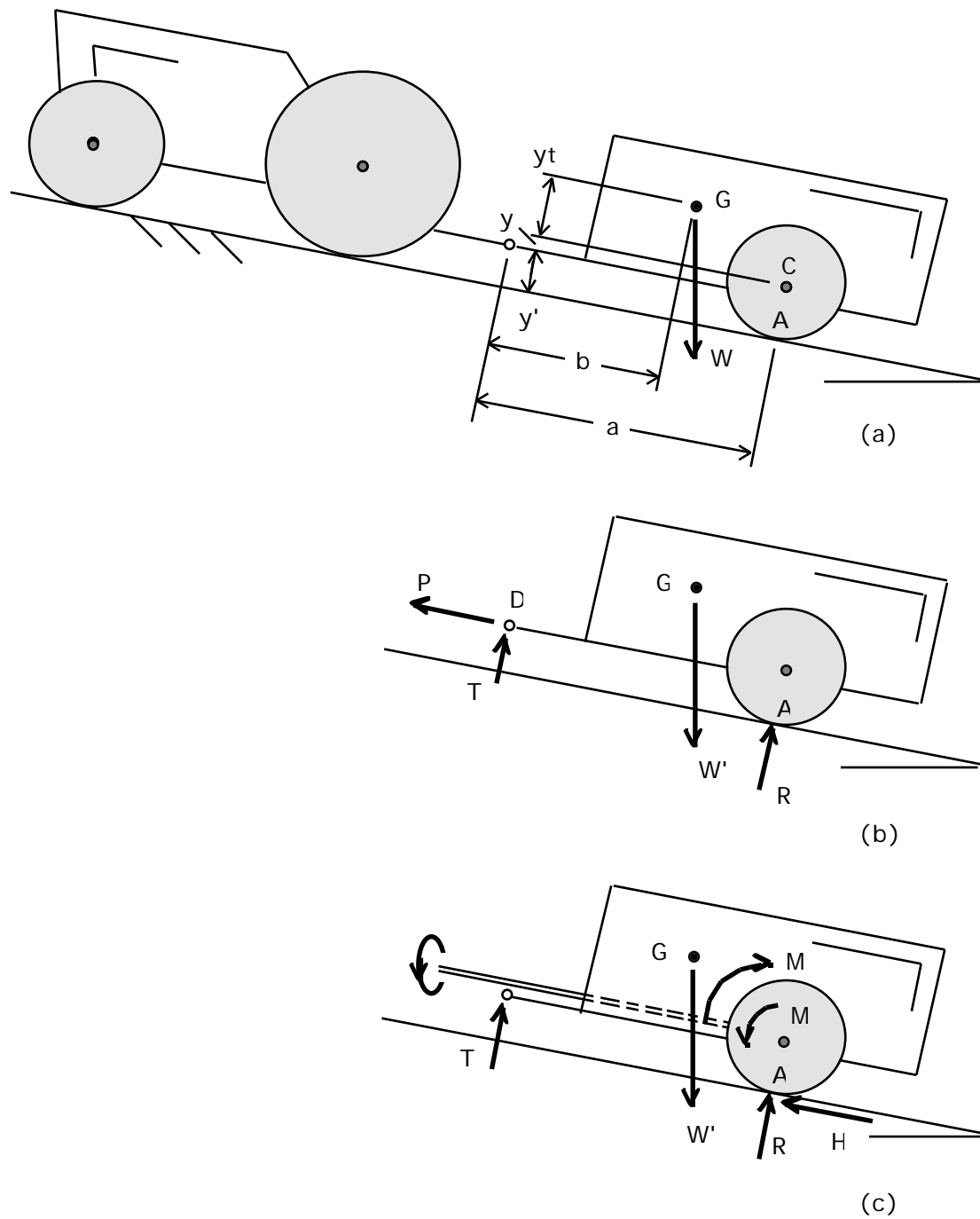


Figure 6.14: Details of PTO driven trailer for analysis

(b) PTO driven trailer

The PTO driven trailer as shown in Figure 6.14 (a) can be pulled up a slope by a tractor (6.14 (b)) or the wheels can also be driven via a drive shaft from the PTO, (6.14(c)).

(i) *Pulled*

Consider the trailer being pulled up the slope as in Figure 6.14(b).

Resolve along the slope for the trailer:

$$P = W' \sin$$

Moments about D for the trailer:

$$R \cdot a = W' \cos b + W' \sin (y+y_t)$$

$$R = W' \cos \frac{b}{a} - W' \sin \frac{y+y_t}{a}$$

Resolve perpendicular to the slope for the trailer:

$$R + T = W' \cos$$

Substitute for R:

$$\begin{aligned} T &= W' \cos - W' \cos \frac{b}{a} + W' \sin \frac{y+y_t}{a} \\ &= W' \cos \frac{a-b}{a} + W' \sin \frac{y+y_t}{a} \end{aligned}$$

(ii) *Driven*

Consider now the wheels being driven so that the drawbar pull on the tractor is brought to zero as in Figure 6.14(c). Determine the tractive coefficient required for the trailer wheels.

Moments about C for the trailer:

$$W' \sin y_t + T \cdot a + M = W' \cos (a-b)$$

Moments about C for the trailer wheels:

$$M = H \cdot r'$$

Resolve along the slope:

$$H = W' \sin$$

Resolve perpendicular to the slope for the trailer;

$$R + T = W' \cos$$

Substitute for T and M above

$$W' \sin y_t + (W' \cos - R) a + W' \sin r = W' \cos (a-b)$$

$$R = W' \sin \frac{r+y_t}{a} + W' \cos \frac{b}{a}$$

$$\mu = \frac{H}{R} = \frac{W' \sin}{W' \sin \frac{r+y_t}{a} + W' \cos \frac{b}{a}} = \frac{a \tan}{(r+y_t) \tan + b} \quad (6.26)$$

The required tractive coefficient thus depends in a complex way on the slope angle and the position of the wheels and the centre of gravity.



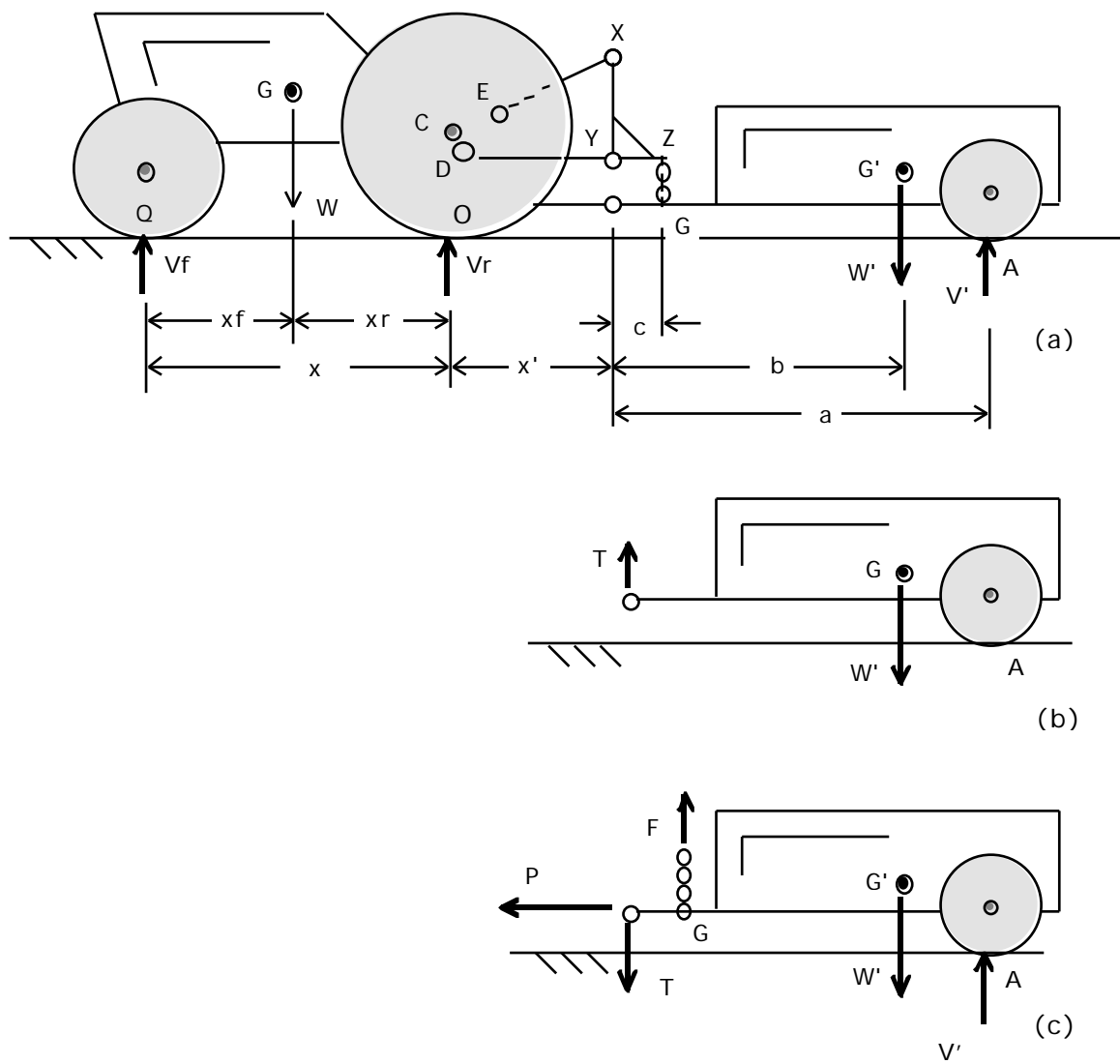


Figure 6.15: Trailed weight transfer hitch; (a) and (b) without lift; (c) with lift

(c) Trailed implement weight transfer system

It was shown above that the traile hitch is least effective in terms of weight transfer. This deficiency has been overcome by the development of a weight transfer hitch, in which part of the weight of trailed implements, and / or downward soil forces, are supported by the tractor rear wheels (Persson, 1967; Hockey, 1961-62).

The principle of one common system is illustrated in Figure 6.15. The rigid link XYZ is attached to the three-point linkage DY and EY; Z is connected to the implement drawbar with a flexible link ZG. In operation, the three-point linkage applies a lifting force F to the implement; this is set by the operator and is kept constant by a hydraulic valve even when the tractor pitches with respect to the implement. This support (but not lifting movement) of the implement transfers some implement weight, as well as some of the tractor front wheel weight, onto the rear wheels.

Assume the weight transfer hitch is attached to an unbalanced trailer as shown in Figure 6.15.

It is required to determine the weight on the rear wheels of the tractor when there is a force F in the chain between the hitch and the drawbar of the trailer. Assume a drawbar pull of P.

(i) *For the tractor and trailer with no lift and no drawbar pull; Figure 6.15(a) and (b).*

Moments about A for the trailer:

$T a = W' (a-b)$  where T is the vertical force on the tractor drawbar

$$T = W' \frac{a-b}{a}$$

Moments about O for the tractor:

$$\begin{aligned} V_f x + T x' &= W x_r \\ V_f &= W \frac{x_r}{x} - T \frac{x'}{x} = W_f - T \frac{x'}{x} \end{aligned} \quad (6.27)$$

Substituting for T from above

$$V_f = W_f - W' \frac{(a-b) x'}{a x} \quad (6.28)$$

Moments about Q for the tractor

$$\begin{aligned} V_r x &= W x_f + T (x + x') \\ V_r &= W_r + T \frac{x + x'}{x} \end{aligned}$$

Substituting for T from above:

$$V_r = W_r + W' \frac{(a-b)(x+x')}{a x} = W_r + W' \frac{a-b}{a} + W' \frac{(a-b) x'}{a x} \quad (6.29)$$

The significance of the terms in Equations 6.28 and 6.29 can be identified as follows:

$$W' \frac{a-b}{a} = T \quad \text{- weight from the trailer drawbar}$$

$$W' \frac{a-b}{a} \frac{x'}{x} \quad \text{- weight from tractor front wheels due to T}$$

(ii) For the tractor and trailer with pull  $P$  and lift force  $F$ , Figure 6.15(c)

Moments about A for the trailer

$$W' (a-b) + P y' + T a = F (a-c)$$

where  $T$  is vertical force on the tractor drawbar and is now assumed to act downwards on the trailer drawbar.

$$T = F \frac{a-c}{a} - W' \frac{a-b}{a} - P \frac{y'}{a}$$

Moments about Q for the tractor

$$V_r x + T (x + x') = W x_f + P y' + F (x+x'+c)$$

$$V_r = W_r + P \frac{y'}{x} + F \frac{x+x'+c}{x} - T \frac{x+x'}{x}$$

Substitute for  $T$  from above:

$$\begin{aligned} V_r &= W_r + P \frac{y'}{x} + F \frac{x+x'+c}{x} - F \frac{(a-c)(x+x')}{a \cdot x} + W' \frac{(a-b)(x+x')}{a \cdot x} + P \frac{y'(x+x')}{a \cdot x} \\ V_r &= W_r + W' \frac{(a-b)(x+x')}{a \cdot x} + P \frac{y'(a+x+x')}{a \cdot x} + F \frac{c(a+x+x')}{a \cdot x} \end{aligned} \quad (6.30)$$

#### Problem 6.15

Show that the weight on the front wheels of the tractor with weight transfer hitch is:

$$V_f = W_f - W' \frac{(a-b)x'}{a \cdot x} - P \frac{(x'+a)y'}{a \cdot x} - F \frac{c(a+x')}{a \cdot x} \quad (6.31)$$

The terms in these equations showing the weight transferred to the rear tractor wheels can be identified as follows:

$W_r, W_f, W'$	- static weight on the respective wheels
$W' \frac{a-b}{a} = T$	- weight on the trailer drawbar
$W' \frac{a-b}{a} \frac{x'}{x}$	- weight transferred from tractor front wheels due to $T$
$P \frac{y'}{a}$	- weight from trailer wheels due to $P$
$P \frac{y'}{x}$	- weight from tractor front wheels due to $P$
$P \frac{y'}{a} \frac{x'}{x}$	- weight from tractor front wheels due to transfer from trailer wheels
$F \frac{c}{a}$	- weight from trailer wheels due to $F$
$F \frac{c}{x}$	- weight from tractor front wheels due to $F$
$F \frac{c}{a} \frac{x'}{x}$	- weight from tractor front wheels due to transfer from trailer wheels

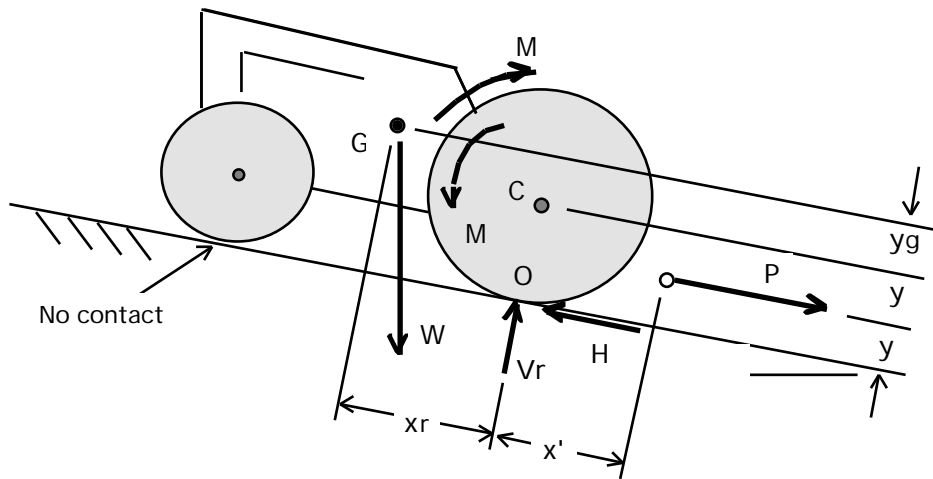


Figure 6.16: Operating parameters for tractor on slope with impending instability

### 6.5 IMPENDING INSTABILITY

The following analysis, which is similar to that given by Sack (1956), illustrates the factors which limit the operation and performance of the tractor as a result of impending instability in the vertical longitudinal plane.

Consider the four-wheel tractor on the slope with drawbar pull parallel to the ground surface;  $\delta = 0$ , as shown in Figure 6.16.

For impending instability, ie,  $V_f = 0$

Moments about C for the tractor:

$$M + W \sin \gamma_g = W \cos \gamma_g x_T + P y$$

Resolve perpendicular to the slope:

$$V_r = W \cos \gamma_g$$

Resolve parallel to the slope:

$$H = P + W \sin \gamma_g$$

Take moments above C for wheel:

$$M = H r$$

Write

$$H = \mu' V_r \text{ where } \mu' \text{ is the gross tractive coefficient (ie, based on } H \text{)}$$

Substitute for H, P and M above:

$$M = \mu' W \cos \gamma_g r$$

Substitute for H, P and M above:

$$W \cos \gamma_g r = W \cos \gamma_g x_T + \mu' W \cos \gamma_g y - W \sin \gamma_g y - W \sin \gamma_g y_g$$

$$\sin (\gamma + \gamma_g) = \cos (\gamma_g x_T + (y - r))$$

$$\tan (\gamma + \gamma_g) = x_T - (r - y)$$

$$\tan (r - y' + \gamma_g) = x_T - (r - y)$$

$$\mu' = \frac{x_T - \tan (r - y' + \gamma_g)}{y'}$$

$$\mu' = \tan \gamma_g + \frac{x_T - \tan (r + \gamma_g)}{y'} \quad (6.32)$$

Dividing through by  $(r + y_g)$  gives

$$\mu' = \tan \gamma_g + \frac{\frac{x_T}{r + y_g} - \tan \gamma_g}{\frac{y'}{r + y_g}}$$

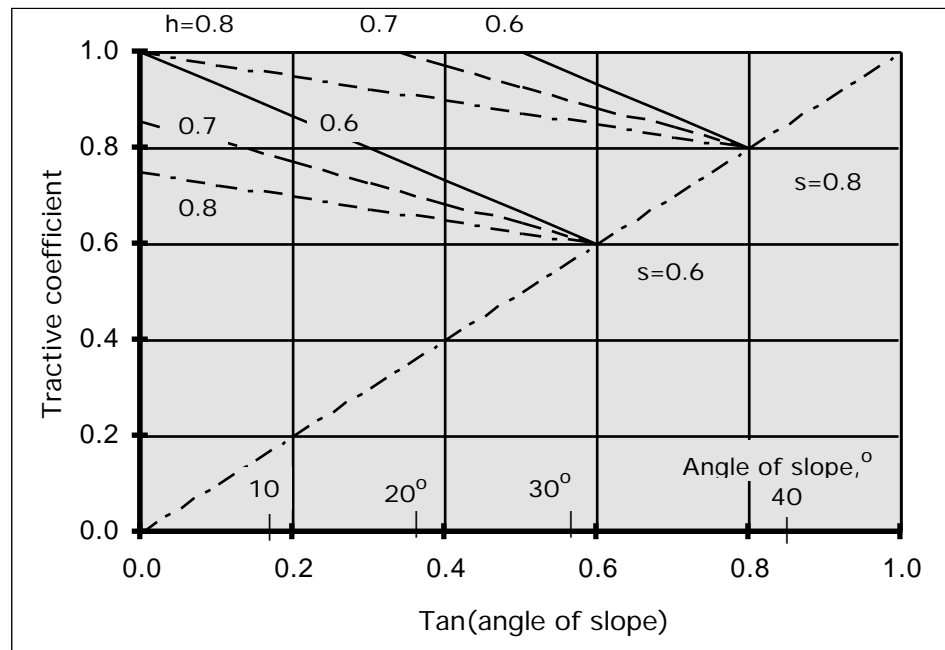


Figure 6.17: Relationships for tractor with impending instability

$$\text{Write } \frac{x_r}{r + y_g} = \tan \theta_{\text{static}} = \tan \theta_s \quad (6.33)$$

$\theta_s$  is the angle of slope that would cause the tractor to tip as a rigid body about the ground contact points under static, ie, no drawbar pull conditions;  $\theta_s$  is usually a large angle, about  $40^\circ$  for most tractors.

$$\text{Write } \frac{y'}{r + y_g} = \frac{\text{drawbar height}}{\text{centre of gravity height}} = h \quad (6.34)$$

A typical value for  $h$  is 0.6.

$$\mu' = \tan \theta_s + \frac{\tan \theta_s - \tan \theta}{h} \quad (6.35)$$

Here  $\mu'$  is the tractive coefficient that must be achieved to bring the tractor to impending instability when it is operating on a slope  $\theta$ .

- (i) If  $\mu'$  required to travel up the slope is less than  $\mu'$  given by Equation 6.35, then the tractor will not reach impending instability.
- (ii) If  $\mu'$  required to travel up the slope is greater than  $\mu'$  given by Equation 6.35, then the tractor will reach impending instability.
- (iii) If  $\mu'$  required to travel up the slope is greater than the maximum  $\mu'$  possible, then the tractor wheels will slip.

Figure 6.17 shows a plot of  $\mu'$  versus  $\tan \theta$  for various values of:

- (i)  $\tan \theta_s = 0.6$  (high centre of gravity) and  $0.8$  (typical centre of gravity)
- (ii)  $h = 0.6$  (typical drawbar height),  $0.7$  and  $0.8$  (a high and dangerous hitch point).

The region where  $\tan \theta > \mu'$  is not feasible; the tractor will slide off the slope.

The example shows a tractor on the slope where  $\tan \theta = 0.3$ .

- (i) For  $\mu'_{\text{max}} = 0.8$  (good traction conditions) instability can occur for  $h = 0.7$  or  $0.8$  because  $\mu'_{\text{max}}$  is greater than  $\mu' = 0.72$  or  $0.67$  required.
- (ii) For  $\mu'_{\text{max}} = 0.6$  (moderate traction conditions) instability cannot occur even for  $h = 0.8$  because  $\mu'_{\text{max}}$  is less than  $\mu' = 0.67$  required; the wheels will slip.

The general conclusion to be drawn is that impending instability:

- (i) is unlikely to occur with normal drawbar heights, moderate slopes and common traction conditions; usually the wheels slip
- (ii) may occur (often with fatal consequences) where traction conditions are good or have been enhanced by the use of strakes (traction aids), where slopes are steep and particularly where the drawbar or the loading point has been raised.

It should also be noted that, while the above simple, static analysis suggests the tractor is relatively safe if used correctly, in practice dynamic effects may influence its behaviour and create dangerous situations. For example, acceleration of the tractor forwards introduces an inertia force through the centre of gravity that has a moment about the rear axle which tends to tip the tractor rearwards. The opposite will be true when the tractor is being braked; here weight is removed from the rear wheels which may adversely affect their braking capacity.

**Problem 6.16**

What drawbar height will just bring the Farmland tractor to impending instability at the maximum tractive force on a horizontal soil surface? The following additional data apply:

Total area of soil - wheel contact patch	A	= 0.076 sq. m;
Cohesion of soil	c	= 2.0 kPa
Angle of internal friction of soil		= 32°

From Figure 6.15:

For impending instability:  $V_f = 0$

Resolving horizontally:  $D = H = A c + W \tan$

Moments about O:

$$H y' = W x_r$$

$$H = W \frac{x_r}{y'}$$

$$W \frac{x_r}{y'} = A c + W \tan$$

$$y' = \frac{W x_r}{A c + W \tan}$$

This confirms the conclusion given above that, as the soil becomes stronger, (c and  $\tan$  increase) the height of the drawbar pull that is required to cause impending instability decreases, ie, instability is more likely under good traction conditions than under poor when the wheels will slip rather than the tractor to tip.

For the Farmland tractor:

$$y' = \frac{W x_r}{(A c + W \tan)} = \frac{2850 \times 9.81 \times 0.54}{.076 \times 2 \times 2000 + 2850 \times 9.81 \times \tan 32} = 0.85 \text{ m}$$

There are places on many tractors at this height to which a load could be attached; it is clearly very dangerous! Loads should always be attached to the standard drawbar.

**Problem 6.17**

Repeat Problem 6.16 for the tractor: (i) travelling up a slope  
(ii) travelling down a slope



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# **The Mechanics of Tractor - Implement Performance**

**Theory and Worked Examples**

**R.H. Macmillan**

**CHAPTER 7 and 8**

**TRACTOR - IMPLEMENT MATCHING AND OPERATION**

**GENERAL PROBLEMS**

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**Note: The Title Page, Preface, Table of Contents, Index, Appendices and details of the Farmland tractor can be found with Chapter 1.**

## CHAPTER 7

### TRACTOR - IMPLEMENT MATCHING AND OPERATION

#### 7.1 INTRODUCTION

Having considered the performance of the tractor on a firm surface (Chapter 3), on soft soil (Chapters 4 and 5) and the effect of the implement on the weight carried by the tractor wheels (Chapter 6), we finally need to consider the steps involved in matching, in performance terms, an implement and tractor. Here matching means choosing the size and / or setting up the tractor and implement so that they may perform their functions in the most efficient way. We will do this by considering what measures would be most appropriate to represent their performance efficiency and how these might be maximised.

Many other factors of an agronomic, economic and organisational nature may also need to be considered, particularly when choosing a type of implement; these are beyond the scope of this book. Readers are referred to existing text books for consideration of the functional performance of the various types of implement.

There may also be many gaps in the information required for matching an implement and tractor. Notwithstanding this lack, setting out the steps in a formal way may help to clarify the logic of making the choices and to determine what further data are required in any particular circumstances.

#### 7.2 IMPLEMENT PERFORMANCE

##### 7.2.1 Implement Draught

The study of the in-field performance of a tractor is related to the performance of the implement to which it is attached; the latter, which is a complex subject in its own right, is also beyond the scope of this book. However we can consider the input to implements in a general way (in terms of force, speed, power etc) and so consider the matching problem, at least in principle.

For the purposes of matching an implement that is being pulled, the important parameter to consider is the horizontal force to move the implement commonly known as the 'draught' force (from the word to 'draw' or to 'pull'). This force is equal and opposite to the forces that arise from the process that the implement is performing and will of course vary with the nature of that process (represented broadly by the implement type), the size of the implement and the travel speed.

If the total force of the implement on the tractor is not horizontal, the vertical component will alter the weight on the wheels, as discussed in Chapter 6, and so affect the traction process. However it will not significantly alter the draught and will only have a second order effect on the matching process.

The draught of an implement is expressed as a force, usually in kN. However draught may also be expressed in terms of parameters that take into account the size of the implement or the magnitude or intensity of the process or of the work that is being done.

These parameters, which are usually termed 'unit draught' or 'specific draught', include:

- (i) draught per unit effective width of machine, kN/m
- (ii) draught per unit of effective cross-sectional area disturbed (usually for tillage implements), kN/sq m (kPa)
- (iii) draught per unit tool (usually for tillage implements), kN/ tool.

These measures are used as a basis for comparing implements of different size and type (ASAE 1998).

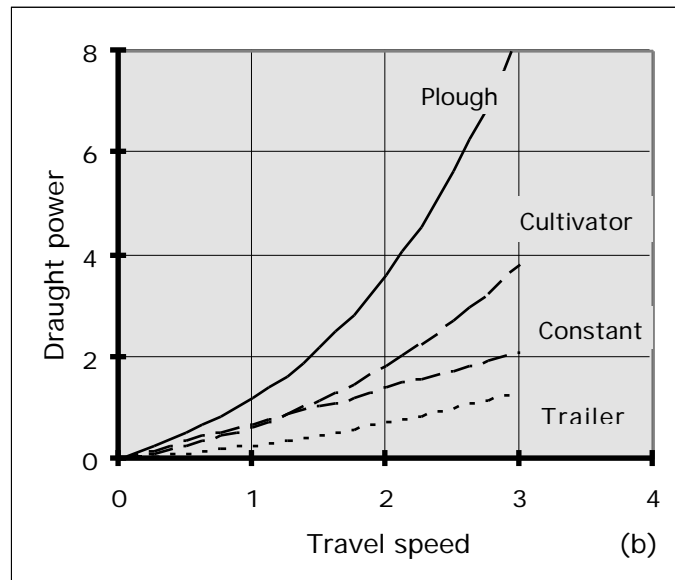
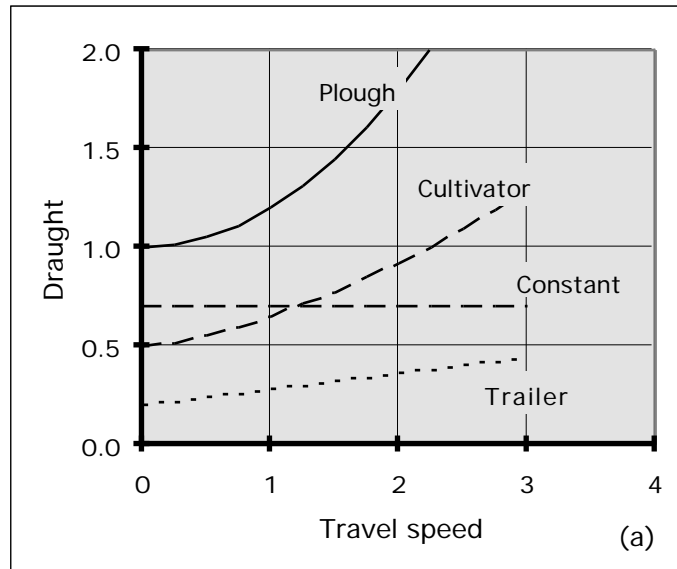


Figure 7.1: (a) Hypothetical draught - speed characteristics for implements, also constant draught  
(b) Corresponding power - speed characteristics

### 7.2.2 Implement draught - speed characteristic

Because the tractor is a variable speed machine, the fundamental and important characteristic of any implement that will be attached to it, (working in given conditions (eg, crop / soil) and with a given adjustment (eg, depth)), is the relationship between its draught force and travel speed.

Some implements, such as those used for tillage, have a significant draught component at 'zero' speed; this represents the force to rupture the soil under 'quasi-static' (ie, effectively zero speed) conditions. At higher speeds the force will generally increase due to the fact that higher speeds involve greater acceleration of the soil and that soils are slightly stronger under dynamic conditions. Hence implements such as mouldboard ploughs that lift and move soil a greater distance and have large draught due to friction and adhesion show a greater increase in draught with speed than do implements, such as tined cultivators, which just lift or move the soil a short distance.

Heavy load carrying implements such as trailers will, due to their rolling resistance, also have a large drawbar pull at zero speed; this may increase slightly as speed increases to moderate levels. For other relatively light implements involved in some form of crop processing (mowing, harvesting, spreading) with power transmitted through the PTO, the draught will be small at zero speed and substantially constant. However the PTO power may increase significantly with speed.

These characteristics of agricultural implements contrast with those of barges being pulled through water where, ideally at least, the draught will be zero at zero speed and vary as the square of the speed (for low speeds).

Figure 7.1(a) shows the hypothetical draught - speed characteristics for various implements also for an implement with a constant draught.

### 7.2.3 Implement power

While draught is the fundamental measure of input to the implement (as drawbar pull is for the output of the tractor), so draught power is a useful measure of input to the implement (as drawbar power is for the output of the tractor).

Since drawbar power is the product of draught force and travel speed, any increase in speed of an implement will cause an increase in the drawbar power due to:

- (i) the direct effect of the travel speed increase
- (ii) the indirect effect due to the associated increase in draught (if any) with increase in travel speed

Thus if an implement has a draught - speed characteristic of the form,

$$D = D_0 + d' \cdot V^{1.2} \quad (7.1)$$

$$\begin{aligned} \text{Draught power } Q &= D \cdot V \\ &= D_0 \cdot V + d' \cdot V^{2.2} \end{aligned} \quad (7.2)$$

The corresponding power - speed characteristics for the implements are shown in Figure 7.1(b). The power - speed characteristic for an implement with a constant draught (for  $d' = 0$  in Equation 7.1) is also shown.

### 7.2.4 PTO driven and towed implements

Many agricultural machines used for 'processing' crop or soil are driven through the PTO as well as being pulled by the drawbar. The effect on the tractor engine will be the sum of the two separate effects.

The increase in engine power required from the tractor with an increase in travel speed (for a constant PTO speed which is typical for a processing type operation) will be the sum of the:

- (i) direct effect on drawbar power of the travel speed increase (as in (i) above) - (first term in Equation 7.2)
- (ii) indirect effect of drawbar power due to the associated increase in draught with travel speed as in (ii) above - (second term in Equation 7.2); this is likely to be small for processing type implements
- (iii) increase in the PTO power due to the increase in rate of crop / soil processing that arises from the increased travel speed.

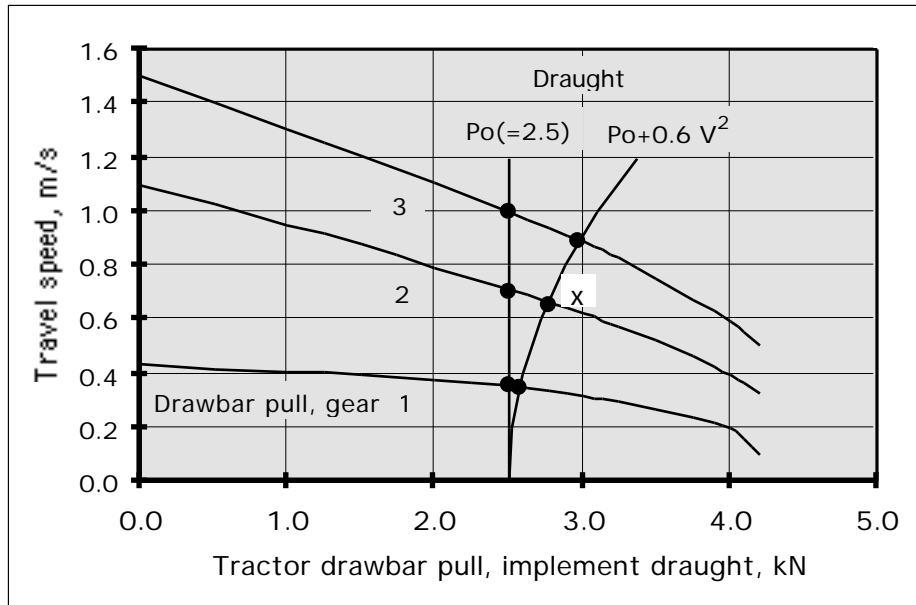


Figure 7.2 Operating points for a tractor in three gears with two implement characteristics

### 7.3 TRACTOR - IMPLEMENT PERFORMANCE

#### 7.3.1 Operating conditions

When an implement is hitched to a tractor:

- (i) the draught of the implement determines the drawbar pull required to be developed by the tractor and is equal but opposite to it
- (ii) the travel speed of the tractor determines the travel speed of the implement and is equal to it

Consider a tractor with a travel speed - drawbar pull characteristic (in a particular gear) attached to and pulling an implement with a particular draught - travel speed characteristic.

Because (i) and (ii) above are true, the operating point of the combination will be where these two curves intersect.

Similarly consider a small tractor with three gears as shown in Figure 7.2 to which we can attach two alternative implements (with a particular width), one with a constant draught - speed characteristic and one where the draught is a function of travel speed,  $V$ . These may be expressed either on an absolute (kN) or on a unit basis (kN / m width).

$$\text{Draught, } D = 2.5$$

$$\text{Draught, } D = 2.5 + 0.6 V^2$$

We can identify the operating conditions for the tractor in the various gears as the points where the draught - travel speed graph for the implement and travel speed - drawbar pull graphs for the tractor intersect as shown in Figure 7.2 at points similar to X. The graph shows the performance for maximum governor setting but it should be remembered that for each gear there is a range of engine governor settings giving a range of lower travel speeds.

We can also imagine the operating points for an implement of different widths having a proportional increase or decrease in draught.

Hence in matching an implement and tractor, there are a large number of possible operating conditions as represented by all the possible intersection points within the overall performance envelope. The question therefore arises as to which point or group of points would represent suitable operating conditions. As an example we might consider a wide implement with the tractor travelling slowly or a narrow implement with the tractor travelling quickly. Within reasonable limits either of these possibilities, or any others between them, would be suitable. However if we wish to consider the efficiency or other aspects of the processes we need to consider the criteria by which we might make a choice between these various alternatives.

#### 7.3.2 Optimum performance criteria

At its simplest the matching process involves deciding what draught to apply on the tractor, ie, what drawbar pull it will be required to develop.

We could consider choosing an implement with a draught that would cause the tractor to reach:

##### (a) Maximum drawbar pull

This (or near it) may be an appropriate condition if we were, for example, attempting to pull a tree over where we could accept a very low travel speed and a very large wheel slip for a few seconds. It would not however be suitable for the long-term continuous operation of a tractor / implement system.

##### (b) Minimum drawbar specific fuel consumption

This would give the best fuel economy. It could be a suitable basis for selection because (at least on firm surfaces) it corresponds to a drawbar power slightly less than the maximum.



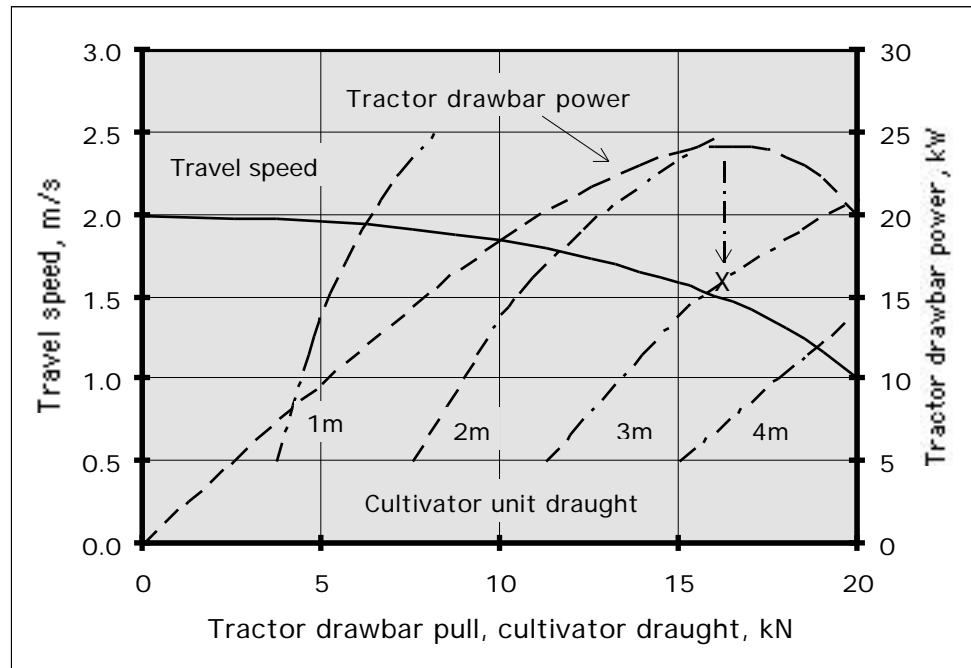


Figure 7.3: Tractor and implement performance for Problem 7.1

(c) Maximum drawbar power

This would also be a suitable basis for selection because maximum power (or a slightly lower value that would allow for natural variation in the draught) would correspond to operation with good fuel economy.

(d) Maximum tractive efficiency

This would also be a suitable basis but with this criteria, drawbar power may be somewhat less than the maximum in (c) above; see for example Figure 5.7 and 5.8.

The most common criterion for optimum matching is that of maximum drawbar power which gives a good fuel economy (criterion (c)) and also a good tractive efficiency (criterion (d)).

**Problem 7.1**

Figure 7.3 shows:

- (i) the travel speed - drawbar pull graph tractor operating in a certain gear on a soil surface.
- (ii) the unit draught (per metre of width) - travel speed graph for a plough cultivating the same soil.

Determine a suitable width for the implement, 1, 2 or 3 m, etc.

Answer:

(a) Tractor

A suitable width implement will be such that the tractor is working at maximum drawbar power. For points on the travel speed - drawbar pull graph, calculate the drawbar power and plot the resulting points against drawbar pull.

For example, with drawbar pull  $P = 5 \text{ kN}$ , travel speed  $V = 1.95 \text{ m/s}$

$$Q = P \cdot V = 5 \times 1.95 = 9.75 \text{ kW}$$

From this graph it is seen that the maximum drawbar power of 24.3 kW will be generated when the drawbar pull is 16.5 kN.

(b) Implement

The draught for the implement will be the width times the draught per metre of width; use this to plot the draught for various widths of implements (2, 3 and 4 m) as shown.

(c) Implement - tractor combination

The operating point for the implement - tractor combination, for maximum drawbar power will be the intersection point of the draught - speed graph (for the particular width of implement) and the drawbar pull graph for the tractor. The point on the graph for the width that intersects at or below 16.5 kN is 3 metres. The implement width is 3m, the drawbar pull is 15.7 kN and the drawbar power is 24 kW.

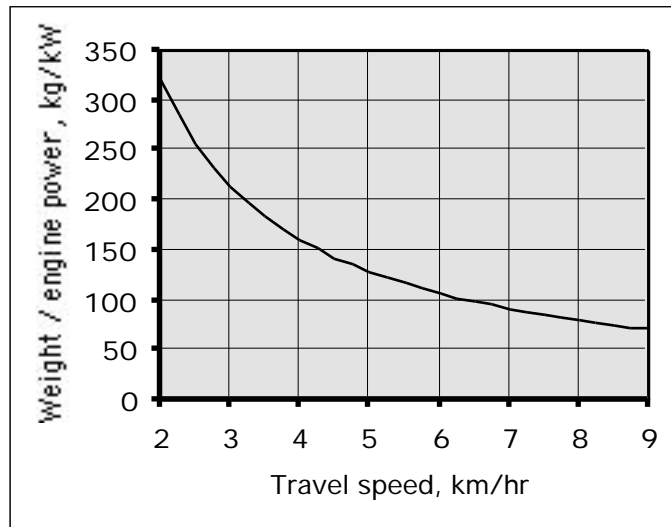


Figure 7.4: Weight - speed relationship for maximum drawbar power; plotted from Dwyer (1984)

### 7.3.3 Matching wheels and engine

Before considering the overall problem of matching implement and tractor it is helpful to consider the more limited design problem of choosing the maximum weight on the driving wheels to suit a given travel speed and engine power.

Neglecting the rolling resistance and the transmission efficiency, the drawbar power for a tractor (Equations 4.22) may be written:

$$\begin{aligned}
 Q_e \eta_t &= Q_d \\
 &= V (Ac + W \tan \phi) X \\
 &= V W \left( \frac{c}{W} + \tan \phi \right) X \\
 &= V W
 \end{aligned} \tag{7.3}$$

This suggests that there should be an inverse relationship between the weight on the driving wheels and travel speed if the maximum tractive power is to be maintained.

Dwyer (1984) gives typical values for maximum tractive efficiency,  $\eta_t = 0.7$  and corresponding tractive coefficient,  $\mu = 0.4$  for a range of tyres and soil conditions.

Hence

$$Q_e 0.7 = V W 0.4$$

$$\frac{W}{Q_e} V = 1.75$$

For  $W$  in kg,  $V$  in km/hr and  $Q_e$  in kW we have

$$\frac{W}{Q_e} V = \frac{1.75 \times 3.6 \times 1000}{9.8} = 643 \tag{7.4}$$

As plotted in Figure 7.4, this shows the inverse relationship between the weight on the driving wheels per kW of engine power and travel speed for maximum performance. This is an important conceptual relationship that illustrates the alternatives of light, 'high' speed tractor / implement systems compared to heavy, slow ones.

For a multi-purpose tractor (with a given engine power) one would choose the lowest (or highest) sensible working speed and calculate the appropriate weight. At higher (lower) speeds the tractor would be heavier (lighter) than required; some weight could be removed (added) if desired.

## 7.4 MATCHING TRACTOR AND IMPLEMENT

### 7.4.1 Variables available

The selection of a tractor / implement system involves a series of choices about the relevant factors. These may be listed as follows:

Purchase	Indirect	Combined	Operation
Maximum engine power	Implement type Implement depth Soil condition	Implement width Weight on wheels	Engine speed Gear ratio

Table 7.1 Parameters in selection, matching and operation of a tractor - implement system

(a) Purchase

This implies the factor is chosen at purchase. Maximum engine power is an upper bound value, the choice of which is a very important one since it determines the maximum capacity of all the equipment that will be used with the tractor. However the issues involved in the choice of an optimum value for it, such as the relative costs of capital and labour and the timeliness penalties or costs of the various operations for which it will be used, are beyond the scope of this book. The following discussion has therefore been limited to the matching of an implement to a tractor that has already been chosen.

(b) Indirect

This implies that these parameters are chosen, not in terms of performance but in consideration of other factors such as the functional objective (implement type), agronomic significance (tillage depth) and weather (soil or crop condition). Again these are also beyond the scope of this book and will not be considered further.

(c) Combined

This implies that these parameters are chosen both at purchase and before operation, ie, they may be altered, in principle at least, but in practice may not be.

(d) Operation

This implies that they are primarily chosen during operation, ie, they can be varied by the operator to suit the conditions that partly arise as a result of earlier choices and partly due to particular local physical circumstances such as land form, soil type, crop condition, etc.

### 7.4.2 Optimising performance

On the basis of the above decisions we are left with four factors that will determine the operating point on the drawbar pull - travel speed and travel speed - draught characteristics;

- (i) engine speed, gear ratio and weight on wheels related to the tractor.
- (ii) implement width related to the implement.

Following the discussion in Section 7.3.2 above, let us assume that the desirable matching criteria is to achieve maximum drawbar power. A somewhat lesser value may be chosen as discussed below; the logic of the argument would be the same.

Equation 2.14 gives

$$\text{Maximum drawbar power} = \frac{\text{Maximum engine power}}{\text{Maximum transmission efficiency}} \times \frac{\text{Maximum tractive efficiency}}{\text{Maximum engine power}} \quad (7.5)$$

From this it will be clear that maximum drawbar power will be achieved if the engine can be made to work at its maximum power and the transmission and the tractor wheels can both be made to work at their maximum efficiencies.

Considering each of these terms in turn:

(a) Maximum engine power

As discussed in Section 3.2.2, maximum engine power will be achieved at the maximum governor setting and with a load (torque) that brings the engine to the condition where the fuel pump is just delivering maximum fuel per stroke. As the load on the engine is increased from zero, the engine speed decreases slightly and the governor increases the fuel flow rate to the maximum; this is the condition of maximum engine power. Any further increase in torque will (because of the constant fuel flow) cause a significant decrease in engine speed and a corresponding reduction in engine power.

The condition of maximum engine power will not be directly evident to the operator. The only evidence will be the single speed value corresponding to maximum engine power.

- (i) A higher speed than this will indicate that the engine, while running in the governed range is not delivering maximum power and is not fully loaded.
- (ii) A lower speed will indicate that the engine is running in the full fuel range, is again not delivering maximum power and is therefore 'over' loaded.

If, for some reason, it is thought to be undesirable to run the engine at full power (eg, to ensure greater engine life), a lesser value of say 90% of maximum power and / or a governor setting less than the maximum setting may be chosen. Such a value would coincide with a general area of good fuel economy and would allow a margin for the load to increase temporarily without the engine running into the full-fuel range.

Strategies to increase (or decrease) the torque on the engine and so bring it to maximum power involves:

- (i) using a higher (lower) gear, ie, decreasing (increasing)  $q$  in Equation 2.2
- (ii) using a wider (narrower) implement, ie, increasing (decreasing)  $P$  in Equation 2.2

(b) Transmission efficiency

As noted in Section 2.4.1(b) above, the transmission efficiency is high and sensibly constant; the operator cannot increase it, so it does not enter into the matching process.

(c) Maximum tractive efficiency

Maximum tractive efficiency will be achieved, by the appropriate choice of implement width (in effect, drawbar pull or strictly draught) and the size and weight on the wheels. However again the condition of maximum tractive efficiency will not be directly evident to the operator, hence it is necessary to use a surrogate variable, ie, one, the value of which, at maximum tractive efficiency, is known; wheel-slip is the variable that may be used.

- (i) too high a slip indicates that the tractor has too large a draught load or has insufficient weight on the driving wheels
- (ii) too low a slip indicates that the tractor has too small a draught load or has excess weight on the driving wheels.

Thus the evidence of the wheels achieving maximum tractive efficiency will be optimum slip. However this varies with the soil condition. Typical values are shown in Table 7.3, adapted from Dwyer et al (1976).

Thus achieving maximum tractive efficiency is based on strategies to decrease (increase) the slip which involves:

- (i) increasing (decreasing) the weight on the wheels
- (ii) using a narrower (wider) implement.

Description of surface	Cone index kPa	Percentage of maximum weight on wheel			
		60 - 70	70 - 80	80 - 90	90 - 100
Dry grass	1500	10	10	10	10
Dry stubble	1000	10	10	11	11
Wet stubble	500	11	12	12	12
Dry loose soil	400	12	13	13	13
Wet loose soil	200	15	16	17	18

Table 7.2 Slip at maximum traction efficiency (Adapted from Dwyer et al, 1976; reproduced with permission of Silsoe Research Institute)

#### 7.4.3 Setting up implement and tractor

It is clear from the above that the optimisation of the performance of a tractor - implement system involves a complex set of choices related to both the engine / transmission and the wheels.

Grevis-James (1978) has developed a grid shown in Table 7.3 that summarizes the changes that may be made to match the tractor and implement and set them up to achieve maximum drawbar power (or some proportion of it).

In this table two alternative strategies are offered.

- maintain the output work rate shown in normal font in the upper part of each cell
- increase the output work rate shown in italic font in the lower part of each cell.

MAINTAIN OUTPUT <i>INCREASE OUTPUT</i>	WHEEL SLIP ->	LOWER THAN OPTIMUM	OPTIMUM	HIGHER THAN OPTIMUM
ENGINE SPEED $\nabla$		WHEELS PART LOADED	WHEELS FULLY LOADED	WHEELS 'OVER' LOADED
HIGHER THAN RATED	ENGINE PART LOADED	Reduce weight, use higher gear & lower governor setting <i>Use higher gear &amp; increase width</i>	Use higher gear & lower governor setting <i>Use higher gear</i>	Reduce width & use higher gear <i>Add weight &amp; use higher gear</i>
RATED	ENGINE FULLY LOADED	Reduce weight <i>Increase width &amp; use lower gear</i>	OPTIMUM MATCHING	Reduce width & use higher gear <i>Add weight</i>
LOWER THAN RATED	ENGINE 'OVER' LOADED	Use lower gear & increase width <i>Use lower gear &amp; increase width</i>	Use lower gear <i>Use lower gear</i>	Reduce width <i>Add weight &amp; use lower gear</i>

- Notes:**
1. Use of a weight transfer hitch or mounted implement has the same effect as adding weight.
  2. "Wheels 'over' loaded" refers to the drawbar load not the weight being carried.
  3. "Engine 'over' loaded" refers to engine running in full - fuel range ( Section 3.2.2, (a) & (b)).
  - 4.. 'Engine speed' refers to rated speed at maximum power, not governor setting.

Table 7.3: Tractor - implement matching chart (Modified from Grevis-James, 1978; reproduced with permission of Institution of Engineers, Australia)

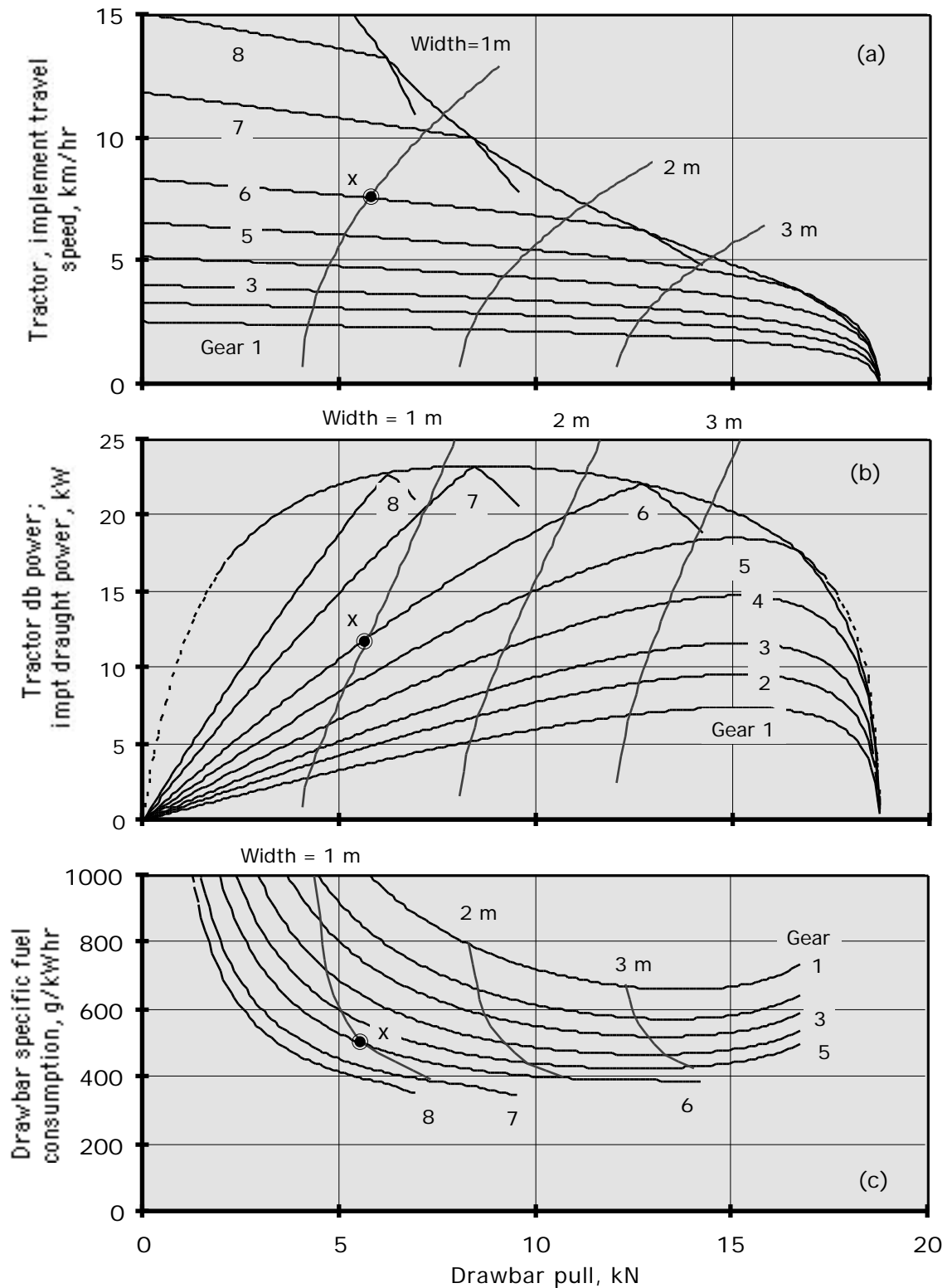


Figure 7.5: (a) Travel speed - drawbar pull and draught characteristic for tractor and implement  
 (b) Drawbar and draught power - drawbar pull and draught characteristic for tractor and implement  
 (c) Specific fuel consumption - drawbar pull and draught characteristic for tractor and implement



## 7.5 OPERATING THE TRACTOR

The above sets out the principles involved in achieving optimum matching of an implement and tractor. In practice the tractor may be used for many different types of work under differing draught and soil conditions. Hence it is unlikely that the tractor / implement will be set up in a way that is optimum for all the types of work for which it may be used.

Notwithstanding this compromise, we need to consider how to adjust the tractor during operation to achieve optimum fuel economy. In doing this the only factors that are available for choice by the operator are the gear ratio and engine speed, as determined by governor setting.

Consider the Farmland tractor and an associated one way plough. The model described in Chapter 5 was used to plot the tractor performance for maximum governor setting for the 6 working gears as shown in Figure 7.5(a).

Figure 7.5 (a) also shows the travel speed versus draught characteristics for three implement widths of 1, 2 and 3 metres (Palmer and Kruger (1982)). This is given by:

$$D = w \cdot d (20 + 0.15 V^2) \quad (7.6)$$

Figure 7.5 (b) shows the drawbar power versus drawbar pull for the 6 gears and draught power versus draught (force) also for same three implement widths; depth = 0.2 m.

The latter is given by:

$$Q = D \cdot V = w \cdot d (20 + 0.15 V^2) V \quad (7.7)$$

Figure 7.5 (c) shows the drawbar specific fuel consumption versus drawbar pull for the 6 gears. The specific fuel consumption graphs for the three implement widths were plotted by projecting down from the appropriate intersection points on the travel speed (or power curves) marked 'X'.

This set of graphs illustrates various aspects of the matching / operation.

- (i) For a given tractor, various implement widths can be used. For heavy work such as ploughing it would be usual to operate in a low gear with an implement that would bring the tractor to near maximum power. This is illustrated by the 3m implement operated by the tractor in 5th gear as shown in Figure 7.5(b).
- (ii) A narrower implement can be operated but, in order to get good fuel economy, it must be worked in the higher gears. This is illustrated by the 2m implement operating in 6th gear as shown in Figure 7.5 (a) and (b).
- (iii) Changing up to a higher gear increases the drawbar power and reduces the drawbar specific fuel consumption along the lines shown in Figure 7.5 (c). Such a change increases the (torque) load on and power from the engine and allows the engine to run in more economical conditions as discussed in Sections 3.2.3, 7.4.2 and 7.4.3.

Changing to a lower gear always makes the fuel economy worse. Clearly the more gears there are available, the smaller will be that change.

- (iv) Changing up a gear increases the speed which may cause control or vibration problems. It may therefore be necessary to reduce the speed by reducing the governor setting; this will mean that the implement characteristic will intersect the characteristic for the chosen gear somewhat below the lines shown at the maximum governor setting in Figure 7.5 (a) and (b).
- (v) If a tractor could be made to work along the maximum power envelope it is clear that the tractor will work in a region of excellent fuel economy. This of course corresponds to the region of high tractive efficiency as shown in Figure 5.9 (c). The limits to this procedure occur:
  - \* at high speeds where rolling resistance power loss is high and ride comfort may be unacceptable
  - \* at high pulls where wheelslip is high.

In summary, when using the tractor for drawbar work, the fuel economy can be improved by changing up a gear and reducing the governor setting, hence the engine speed, to avoid excessive travel speed. If it is necessary to change down a gear, the fuel economy will be worse; increasing the engine speed by increasing the governor setting will improve it to some extent.

If the PTO is being used and a fixed speed for it is required, it would only be possible to change gears; the fuel economy will change as above.

Notwithstanding all of these choices and adjustments, it is important to operate the tractor under safe conditions where control can be maintained and at speeds with which the work being done is satisfactory.

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## CHAPTER 8

### GENERAL PROBLEMS

#### Problem 8.1

A tractor has an engine having a maximum power of 62kW at 1950 rpm at maximum governor setting.

When tested in the field, the following data were obtained:

Drawbar pull	= 26.2 kN	
Distance traveled for 10 revolutions of driving wheels - with no drawbar pull		= 55.8 m
	- with drawbar pull	= 46.2 m
Engine speed	= 1950 rpm	
Fuel consumed	= 126 g	
Time taken	= 25.8 s	
Transmission efficiency	= 92%	

Determine: Drawbar power, wheel slip, traction efficiency, fuel consumption and specific fuel consumption.

Answers: 46.9 kW; 17.2%; 82%; 21L/hr; 374 g/kWhr

#### Problem 8.2

The following data applies to a tractor operating on a level frictional soil:

Diameter of driving wheels	= D
Overall gear ratio	= q
Maximum engine torque	= T
Angle of internal friction	=

Show that the minimum weight on the wheels to bring the engine to maximum torque is given by:

$$W = \frac{2qT}{D \tan \phi}$$

#### Problem 8.3

Consider a rear wheel drive tractor operating on a level surface. If the coefficient of traction based on the weight on the rear wheels is  $\mu$  and the coefficient of rolling resistance for the front wheels is  $r$  show the drawbar pull that can be achieved is:

$$P = \frac{W_r - W_f}{1 - \frac{y}{x} (y + r)}$$

#### Problem 8.4

Consider a tractor with rear wheel braking for which the maximum braking coefficient,  $\mu$  is

$$\mu = \frac{\text{maximum horizontal braking force}}{\text{vertical rear wheel reaction}}$$

By assuming that the dynamic inertia force 'ma' in braking is a static force acting through the centre of gravity of the tractor, show that the maximum retardation, a is given by:

$$a = g \frac{x_r}{x + (r + y)}$$

**Problem 8.5**

A small, rear wheel drive tractor was tested in three gears with normal weight on a bitumen road and on a firm soil surface also with extra weight also the road. The results are shown in Table 8.1.

Drawbar pull, kN	Soil, standard weight				Road, standard weight			
	Travel speed, m/s			Slip %	Travel speed, m/s			Slip %
	3	2	1	%	3	2	1	%
Gear ->								
0.0	1.51	1.05	0.42	0.0	1.63	1.07	0.45	0.0
1.0	1.32	0.95	0.40	3.5	1.54	1.03	0.42	1.7
2.0	1.13	0.83	0.36	9.9	1.44	0.99	0.40	3.6
3.0	0.90	0.68	0.29	18.4	1.35	0.95	0.38	5.9
4.0	0.57	0.44	0.12	-	1.26	0.90	0.35	9.0
5.0	-	-	-	-	1.09	0.83	0.29	14.1
6.0	-	-	-	-	0.60	0.48	-	25.0

Drawbar pull, kN	Road, extra weight				Fuel cons, L/hr: gear 3		
	Travel speed, m/s			Slip	Soil	Road	Road
	3	2	1	%	Std wt.	Std. wt	Extra wt.
Gear ->							
0.0	1.64	1.08	-	0.0	1.80	1.66	1.50
1.0	1.56	1.04	-	1.6	2.20	2.10	1.95
2.0	1.49	0.99	-	3.0	2.50	2.50	2.35
3.0	1.42	0.96	-	4.6	2.65	2.87	2.74
4.0	1.35	0.92	-	6.6	-	3.10	3.10
5.0	1.28	0.88	-	9.1	-	-	3.35
6.0	1.19	0.82	-	12.8			
7.0	0.95	0.70	-	19.8			

**Table 8.1**

(i) Plot:

- Travel speed and wheelslip versus drawbar pull
- Drawbar power versus drawbar pull
- Fuel consumption and specific fuel consumption versus drawbar power for gear 3
- Drawbar power versus wheelslip

(ii) Discuss the effect of gear, weight and surface on the performance of the tractor.

**Problem 8.6**

Some tractors are available with a three-point linkage on the front of the tractor. Compare such an arrangement with a rear-mounted linkage with respect to weight transfer and implement control.

**Problem 8.7**

Imagine that you have been requested to advise on the preliminary design of a harvesting machine to be powered by the Farmland tractor. The machine will be towed over very firm soil by the drawbar and the harvesting mechanism will be driven by the PTO.

The machine has the following performance characteristics:

Specific draught	= 8 kN/m of width
Specific PTO power	= 3.2 kW/m of width
Operating speed not more than	= 3 km/hr (approx.)

Assume the following for the tractor:

Traction efficiency	= 70 %
Transmission efficiency to wheels	= 90 %
Transmission efficiency to PTO	= 90 %
Wheel slip not more than	= 15 %

Using the graphs given in Chapter 3 and considering both power and draught requirements, estimate the maximum width of harvester that can be operated and the fuel consumption.

Answers: 2.25 m, 8 kg/hr

### Problem 8.8

(a) Show that the slope, on which the Farmland tractor will just roll forwards is given by:

$$\tan \theta = \frac{X_{r_r} + X_f}{x + (r + y)(r_r + r_f)}$$

Hence determine the slope for the tractor on:

- (a) concrete
- (b) loose sand

(b) Repeat for the tractor rolling rearwards

### Problem 8.9

By taking appropriate measurements of a small motor bike investigate its capacity to operate a small trailer. Give careful consideration to instability and safety issues.

### Problem 8.10

Apply the principles developed for the two-wheeled tractor in Section 6.4.4 to the design of a trailer of the type shown in Figure 6.1(b) for use with such a tractor.

- X - X - X -